Review Model Problems

Particle—no potential
Wave solution—continuous wavelength/Energy
Since only (T=H) KE _ [*H*,p] = 0
And particle has conserved momentum (Classical no Force)
INSERT EQN

Add potential – E, λ no longer continuous quantization from B.C. (restrict motion)

1.
V = Const. But restricted in space:
-- ∞ box-artificial but good model to illustrate
E – number of wavelengths in box (curvature)
Must be half integral (_ (0)= _(L) =0) so quantize
-- circular box _ same picture but integer wavelength _ continuous first de??? or no double value _(2π)= _(?????
Becomes eigen function Lz
--multidimensional box _ final degeneracy as a function of symmetry
[if x=y then excitation along x and y must have the same impact]
provides great example of separation of variables summed *H*, E _ product _ works because coordinates are independent

Now if potential not infinite still have B.C. lent these are ones related to first postulate in that _ must be contiuous etc., and ∫ _*y = N
Also expect continouity between problem with V = ∞ and V = very large _ steady change as walls???

Atkins 2.3 and 2.5
1. General aspects of Schröedinger's Eqn
INSERT EQN
_ derived to correspond to classical mech, not contain "pure gin" prop such as spin
_ also no relativistic connection
Now looking at time dependence—not a wave eqn
From classical point of view wave equation time/space both second de??? form is diffusion equation
(does tell us what particle does in moving)

for $H \neq H$(t)

Separation: $\varphi(x,t) = \varphi(x)e^{-iEt/K}$

Time indep part: $H\,\varphi(x) = E\,\varphi(x)$ _ standing wave equation

i.e. wave equation from spatial point

Time dep part: just a phase 1 _ i _ 1 _ i _ 1 _????

Trick here is to remember that all measurement use $\varphi^*\varphi$(prob) $\varphi^*\alpha\,\varphi$(prob. meas. α)

So phase cancels: $e^{iEt/K} * e^{-iEt/K} = 1$

So have _ * _ = _ (x)* _(x) ≠ f(t) – stationary state

2. Quantization – big difference from classical – Energy not continous (??)

rearrange: **INSERT EQN** curvature

-- V=E on _ = 0 _ no curvature **INSERT DRAWING**

-- _ ≠ 0, curvature ~ V-E _ sign change:

all of this makes the function oscillate (wave) so the potential – causes the wave behavior but also leads to the bounds because ∫_*_ dT = N

INSERT GRAPH

In order to get the _ _ 0 behavior

Slope at x for V-E ~ must be limited—not all shapes

Only a few will give _ _ 0 both sides potential contains the ???????? particle

Particle in a finite well: **INSERT GRAPH**

INSERT EQN

at boundary Ψ(0), $E < V_0$ so have <u>change</u> from E > V to E < V

– curvature must change at wall from (-) to (+) from Ψ(+)

– Restriction to be finite _ e^{+kx} (x < 0) <u>damp</u>

– in II – Ψ(+), (V – E) _ curvature

INSERT GRAPH

Now at Ψ(L) _ change sign curvature again to (+) quantization _ only contain Ψ(L) values lead to proper damping _ other integral # λ's fit

Y = 0 _ zero curve so opp curve Ψ(+) and Ψ(-)

Putting together the sides of I II, and II III requires finding the <u>amplitude</u> of the function at x = 0 and x = L _ x = 0 : C = A

INSERT EQN

Now what is unusual is that the constants are functions of the energy levels – pib _ only of L

so ratioing **INSERT EQN**

give a complex function that could be solved for E

INSERT EQN

Point: this solved only for certain values of E

Handout

Kangman, p. 191, figure plot both sides find intersection

– regular spacing violated – pib – expand ~k^2, have ~ const (amide)

– number of levels limited: **INSERT EQN** (max level)

– E > V get continuum, unbound states

What about inverting the problem – tunnelling?

INSERT GRAPH

so now particle in Region I can approach II and see a wall (V_0)

INSERT EQN

INSERT EQN

so have plane wave solution in each part:

$\Psi_p = A_p e^{+ikx} + B_p e^{-ikx}$ p = I, II, III

INSERT EQN

INSERT EQN

now if E < V_0 _ classically particle bounces back and does not penetrate barrier (stays on one side)

but k_{II} = imagine = iK **INSERT EQN**

$\Psi_{II} = A_{II} e^{-Kx} + B_{II} e^{+Kx}$

so in barrier _ function decays and grows exp but does not oscillate

as width on height increases – B_{II} _ 0 since clearly Y_{II} _ 0 (inside barrier)

but _ thin barrier not yet zero and if thin enough will be some probability on other side (tunnel)

ex 1eV electron in 2V barrier

decays ~ $e^{-5.12(x/nm)}$ so decays ~1/e in 0.2 nm

Thin barrier $B_{II} \neq 0$ since rising function stays finite inside

X = 0: $A_I + B_I = A_{II} + B_{II}$ $Ae^{ikl} + Be^{-ikl} = A_E e^{-kl} + B_{II} e^{-kl}$

now have 4 eqn and 6 unknowns _ if assume specific case of particle approaching barrier from left (momentum right)

Then know $B_{III} = 0$ (i.e. particles right of barrier only to risk) but cannot set $B_I = 0$ since could reflect.

Now: **INSERT EQN** _ probability of reflection
INSERT EQN _ probability of penetration (tunnel)

Transmission probi/Ag **INSERT EQN** turns out

Correspondence with classical:
How work:
1. as V increase _ G ~ Ve **INSERT EQN** _ ∞ , and P _ 0
So high barrier _ no transmission
2. as L increase _ G ~ e^{KL} _ ∞, P _ 0, so wide barrier _ no transmission
3. note: as m increase, G inc we^{tm-} and P _ 0
4. as E increases _ G ~ **INSERT EQN** _ 0, so very high energy P _ non zero
(eg. Like bullet on better β particle)

so if E suff and V,L small oscillary (line mom.) mave/ part ????? strike wall from left, part penetrate damp, rearrange with reduced amplitude A_{III}

antitunnelling (analogy to reflection when change indep)
for E > V classically P = 1 but with QM P<1, some reflection due to V