1. [7.11(b)] An unnormalized wavefunction for an electron in a carbon nanotube of length L is sin(2πx/L). Normalize this wavefunction.

2. [7.12(b)] For the system described in Exercise 7.11b, what is the probability of finding the electron in the range dx at x = L/2?

3. [7.13(b)] For the system described in Exercise 7.11b, what is the probability of finding the electron between x = L/4 and x = L/2?

4. [7.4] The ground-state wavefunction for a particle confined to a one-dimensional box of length L is

$$\psi = \left(\frac{2}{L}\right)^{1/2} \sin \left(\frac{\pi x}{L}\right)$$

Suppose the box is 10.0 nm long. Calculate the probability that the particle is (a) between x = 4.95 nm and 5.05 nm, (b) between x = 1.95 nm and 2.05 nm, (c) between x = 9.90 nm and 10.00 nm, (d) in the right half of the box, (e) in the central third of the box.

5. [8.6(b)] What are the most likely locations of a particle in a box of length L in the state n = 5?

6. [8.8(b)] A nitrogen molecule is confined in a cubic box of volume 1.00 m³. Assuming that the molecule has an energy equal to 3/2kT at T = 300 K, what is the value of n = (n_x² + n_y² + n_z²)½ for this molecule? What is the energy separation between the levels n and n + 1? What is its de Broglie wavelength?

7. [8.19(b)] Calculate the value of |mₗ| for the system described in the preceding exercise corresponding to a rotational energy equal to the classical average energy at 25°C (which is equal to 1/2kT).

Note: from 8.19(a) the system is a proton constrained to rotate in a circle of radius 100 pm around a fixed point.

8. [8.10] The wavefunction inside a long barrier of height V is ψ = Ne^−kx. Calculate (a) the probability that the particle is inside the barrier and (b) the average penetration depth of the particle into the barrier.
9. [8.14 (a) only] Determine the values of $\Delta x = (\langle x^2 \rangle - \langle x \rangle^2)^{1/2}$ and $\Delta p = (\langle p^2 \rangle - \langle p \rangle^2)^{1/2}$ for (a) a particle in a box of length $L$ and (b) a harmonic oscillator. Discuss these quantities with reference to the uncertainty principle.

10. [8.32] Many biological electron transfer reactions, such as those associated with biological energy conversion, may be visualized as arising from electron tunnelling between protein-bound co-factors, such as cytochromes, quinones, flavins, and chlorophylls. This tunnelling occurs over distances that are oftengreater than 1.0 nm, with sections of protein separating electron donor from acceptor. For a specific combination of donor and acceptor, the rate of electron tunnelling is proportional to the transmission probability, with $\kappa \approx 7 \text{ nm}^{-1}$ (eqn 8.19). By what factor does the rate of electron tunnelling between two co-factors increase as the distance between them changes from 2.0 nm to 1.0 nm?

Extra (do not hand in):

11. [7.8] The normalized wavefunctions for a particle confined to move on a circle are $\psi(\phi) = (1/2\pi)^{1/2} e^{-im\phi}$, where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$ and $0 \leq \phi \leq 2\pi$. Determine $\langle \phi \rangle$.

12. [8.1(b)] Calculate the energy separations in joules, kilojoules per mole, electronvolts, and reciprocal centimetres between the levels (a) $n = 3$ and $n = 1$, (b) $n = 7$ and $n = 6$ of an electron in a box of length 1.50 nm.

13. [8.3(b)] Calculate the expectation values of $p$ and $p^2$ for a particle in the state $n = 2$ in a square-well potential.

14. [8.4(b)] Calculate the expectation values of $x$ and $x^2$ for a particle in the state $n = 2$ in a square-well potential.

15. [8.7(b)] Calculate the percentage change in a given energy level of a particle in a cubic box when the length of the edge of the cube is decreased by 10 percent in each direction.

16. [8.18(b)] Confirm that wavefunctions for a particle in a ring with different values of the quantum number $m_l$ are mutually orthogonal.

17. [8.28] Starting from the operator $\hat{L}_z = xp_y - yp_x$, prove that in spherical polar coordinates $\hat{L}_z = -i\hbar \partial/\partial \phi$. 