EXAMPLE PROBLEM 8.3

An ideal solution is made from 5.00 mol of benzene and 3.25 mol of toluene. At 298 K, the vapor pressure of the pure substances are $P^*_{benzene} = 96.4$ Torr and $P^*_{toluene} = 28.9$ Torr.

a. The pressure above this solution is reduced from 760 Torr. At what pressure does the vapor phase first appear?

b. What is the composition of the vapor under these conditions?

Solution

a. The mole fractions of the components in the solution are $x_{benzene} = 0.606$ and $x_{toluene} = 0.394$. The vapor pressure above this solution is

$$P_{total} = x_{benzene}P^*_{benzene} + x_{toluene}P^*_{toluene} = 0.606 \times 96.4 \text{ Torr} + 0.394 \times 28.9 \text{ Torr}$$

$$= 69.8 \text{ Torr}$$

No vapor will be formed until the pressure has been reduced to this value.

b. The composition of the vapor at a total pressure of 69.8 Torr is given by

$$y_{benzene} = \frac{P^*_{benzene}P_{total} - P^*_{benzene}P^*_{toluene}}{P_{total}(P^*_{benzene} - P^*_{toluene})}$$

$$= \frac{96.4 \text{ Torr} \times 69.8 \text{ Torr} - 96.4 \text{ Torr} \times 28.9 \text{ Torr}}{69.8 \text{ Torr} \times (96.4 \text{ Torr} - 28.9 \text{ Torr})} = 0.837$$

$$y_{toluene} = 1 - y_{benzene} = 0.163$$

Note that the vapor is enriched relative to the liquid in the more volatile component, which has the lower boiling temperature.

EXAMPLE PROBLEM 8.4

For the benzene–toluene solution of Example Problem 8.3, calculate

a. the total pressure

b. the liquid composition

c. the vapor composition

when 1.50 mol of the solution has been converted to vapor.

Solution

The lever rule relates the average composition, $Z_{benzene} = 0.606$, and the liquid and vapor compositions:

$$n_{vapor}(y_{benzene} - Z_{benzene}) = n_{liq}(Z_{benzene} - x_{benzene})$$

Entering the parameters of the problem, this equation simplifies to

$$6.75x_{benzene} + 1.50y_{benzene} = 5.00$$

The total pressure is given by

$$P_{total} = x_{benzene}P^*_{benzene} + (1 - x_{benzene})P^*_{toluene} = [96.4x_{benzene} + 28.9(1 - x_{benzene})] \text{ Torr}$$

and the vapor composition is given by

$$y_{benzene} = \frac{P^*_{benzene}P_{total} - P^*_{benzene}P^*_{toluene}}{P_{total}(P^*_{benzene} - P^*_{toluene})} = \left[\frac{96.4\dfrac{P_{total}}{\text{Torr}} - 2786}{67.5\dfrac{P_{total}}{\text{Torr}}}\right]$$

These three equations in three unknowns can be solved by using an equation solver or by eliminating the variables by combining equations. For example, the

first equation can be used to express $y_{benzene}$ in terms of $x_{benzene}$. This result can be substituted into the second and third equations to give two equations in terms of $x_{benzene}$ and p_{total}. The solution for $x_{benzene}$ obtained from these two equations can be substituted in the first equation to give $y_{benzene}$. The answers are $x_{benzene} = 0.561$, $y_{benzene} = 0.810$, and $P_{total} = 66.8$ Torr.