Section II: BASIC PRINCIPLES OF PULSED NMR

II.1 THE PULSED NMR EXPERIMENT

Our goal is to determine how many spins precess at a frequency \( \omega_0 \) or, equivalently, how many precess at an offset \( \Delta \omega = \omega_0 - \omega \).

A simple way:

i) Let \( M_0 \) form in the presence of \( B_0 \)

\[
\hat{H} = -\gamma B_0 S_z
\]

\[
\hat{H} = -\omega_0 S_z
\]

\[
S_z = S_z, \text{v.n., angular mom.}
\]

\[
\beta = \frac{\omega_0}{\gamma}
\]

\[
M_0 = \alpha z
\]

\[
S_z = \text{spin, angular mom.}
\]

\[
\omega_0 = \beta B_0
\]

\[
M = (0, 0, M_0) \beta = \gamma
\]

ii) Give a short but strong rf pulse in the neighborhood of \( \omega_0 \) along the x axis of the rotating frame: much harder

During this pulse:

\[
\hat{H}_r = -\Delta \omega S_z - \omega_1 S_x \approx -\omega_1 S_x
\]

\[
M = (0, 0, M_0)
\]

\[
\alpha = (0, 0, \alpha_0)
\]

\[
\beta = \omega_1 \cdot \text{pulsed = pulse angle}
\]

\[
\beta = 20\, \text{kHz}
\]

\[
\text{rotates once in 50 ms}
\]
If $\beta = \pi/2$

\[
\mathcal{H} = -\Delta \omega S_z.
\]

After the pulse

\[
U(t) = e^{-i \mathcal{H} t} = e^{-i \Delta \omega S_z t}.
\]

If $\beta = \pi$

\[
\mathcal{H} = S_z.
\]

\[
U(t) = e^{-i \mathcal{H} t} = e^{-i S_z t}.
\]

The Hamiltonian $\mathcal{H}$ is rotation around $z$ at a rate $\Delta \omega$, which can be detected by a pair of coils placed in the lab frame.

Time evolution operator $U(t)$: rotation matrix

**Unitary operator $U(t)$:**

- $M_z(t) = \gamma h a_z = M_0 \cos \beta$
- $M_x(t) = \gamma h a_x = M_0 \sin \beta \cdot \sin(\omega_0 t)$
- $M_y(t) = \gamma h a_y = M_0 \sin \beta \cdot \cos(\omega_0 t)$

\[ (x, y, z) \text{: Larmor rotation} \]
The oscillating magnetization will induce changes in magnetic flux, which will in turn induce emfs (voltages) in coils. The resulting signal $S$

\[ S = \frac{d\phi}{dt} \text{ Faraday's Law} \]

\[ \phi: \text{magnetic flux} \approx 10^{-10}\text{ Wb} \]

\[ M_0: \text{voltage} \]

\[ S \propto M_0 \cdot \omega_0 \cdot \cos(\omega t) \] \text{ Detected in pulsed NMR}

\[ S \propto M_0 \cdot \omega_0 \cdot \sin(\omega t) \] \text{ Detected in} \]

\[ \text{Twice the voltage between 2 \& 1 looked at coils (2 \& 1)} \]

It is convenient to express the rotating magnetization as a single quantity rotating in the complex plane:

\[ M_+ = M_x + iM_y (M_+ = T_x (e^{iS_+}) \text{ in quantum mechanics, } S \leftarrow (0,0)) \]

\[ S_+(t) = S_x + iS_y \cdot M_0 \cdot \omega_0 \cdot e^{i\omega t} \]

Note that $S_+(t) = i \cdot \omega_0 \cdot M_+(t) = \text{constant} \cdot M_+(t)$

It is therefore usually said that NMR detects the magnetization in the x-y plane. Strictly speaking however, there is a $i\omega_0$ factor involved.

Moreover, since $M_0 = N \alpha^2 h^2 B_0 / 4kT$

\[ \Rightarrow S = \frac{\alpha^3 h^2}{4k} \cdot N \cdot B_0^2 \cdot e^{i\omega t} \]

\[ \text{comes from electronics \& all come down to a } \alpha \text{ factor} \]

Dec $\rightarrow$ Inc $\omega_0$

$N \alpha \text{ signal}$

$\rightarrow$ Inc $\omega_0$

one to from $B_0 \rightarrow$ Pinc

Inc $\omega_0$
II.2 QUADRATURE DETECTION IN THE ROTATING FRAME

In pulsed NMR a single coil is used for

Irradiation

detection

The possible coil geometries:

The irradiation pulse: \( B_1 \cos (\omega t + \phi_{Tx}) \)

Tx: Transmitter

The observed signal: \( S_0 \cos (\omega t + \phi_{Tx}) \)

How do we get Imag. part. (before had coil in \( y \))

Only one component \( s(x) \) w/phase info

*Somewhat ineffective
*How do we make a turn \( N \)-N coil?
One can still observe $S_x, S_y$ without using 2 orthogonal coils by using a double balanced mixer (DBM); the use of this approach leads to an experimental scheme called phase-sensitive detection or phase-sensitive demodulation.

A DBM is a device that takes in 2 signals $S_1, S_2$ and gives an output $\phi(t) \propto S_1(t) \cdot S_2(t)$.

A filter is chosen so that fast oscillating terms go away. In phase-sensitive detection experiments, the source used for irradiation is also used to demodulate the signal according to:

![Diagram of DBM circuit]

$$C_5 = \sin(\phi(t))$$

$$S_x = \cos(\omega t + \phi_{RX}) \cos(\omega t + \phi_{TX})$$

$$S_y = [\omega - \omega_0]t + (\phi_{RX} - \phi_{TX}) + \text{fast osc. terms}$$

$$\Delta W = \omega - \omega_0 : \text{offset}$$

$$\phi = \phi_{RX} - \phi_{TX}$$

\[\text{eg: } \omega_0 = 100.01 \text{ MHz, } W = 10.00 \text{ MHz, } \Delta W = 2 \text{ Hz, eliminate by } 1/W + W/2 = 2 \text{ kHz how fast filter?}\]
We can therefore observe the two signals expected from orthogonal coils but in the rotating frame: quadrature detection.

Note that whereas \[ \omega, \omega_0 \approx 100\text{'s MHz (rf)} \]
\[ \Delta \omega \approx 0-100 \text{ KHz (audio)} \]

† All of NMR detection takes place in the rotating frame.
† Strange things happen in rotating frame.
II.3 FOURIER ANALYSIS

Suppose that in the past example \( \theta = 0 \)

\[
S(t) = S_0 \left[ \cos(\Delta \omega t) + i \sin(\Delta \omega t) \right] = S_0 e^{i\Delta \omega t}
\]

In these cases, it is relatively simple to extract \( S_0, \Delta \omega \)

But, what happens if we have a superposition of several waves:

\[
S(t) = f_1 e^{i\omega_1 t} + f_2 e^{i\omega_2 t} + f_3 e^{i\omega_3 t} + \ldots
\]

\[ \omega_1 < \omega_2 < \omega_3 < \ldots ; \quad 0 \leq t \leq T \]

Now we want \( f(\omega) = \begin{cases} 
  f_1 \quad \text{at } \omega_1 \\
  f_2 \quad \text{at } \omega_2 \\
  \vdots 
\end{cases} \]

from the interferogram \( S \):
To extract each of these components, we multiply by $e^{-i\omega t}$ and integrate over $t$. In the case of $\omega_2$ for instance

$$S(t)e^{-i\omega_2 t} = f_1 e^{i(\omega_1 - \omega_2)t} + f_2 e^{i(\omega_2 - \omega_2)t} + f_3 e^{i(\omega_3 - \omega_2)t} + f_4 e^{i(\omega_4 - \omega_2)t} + \ldots$$

The real part:

The integral extracts the amplitude at $\omega_2$

$$\Rightarrow \frac{1}{T} \int_{0}^{T} S(t)e^{-i\omega_2 t} \, dt = \mathcal{F}(\omega_2)$$

In general:

$$\mathcal{F}(\omega_k) = \frac{1}{T} \int_{-T}^{T} S(t)e^{-i\omega_k t} \, dt$$

**Fourier Analysis ($T \to \infty$)**

This Fourier transformation relates the time-domain signal $S(t)$ experimentally available with the desired spectrum $F(\omega)$. 

want $\int_{-\infty}^{\infty}$
II.4 DISCRETE SAMPLING AND FAST FOURIER TRANSFORM

We had demodulated our NMR signal into 2 audio components. These voltages are now sampled at equal time intervals $\Delta t$ by an analog-to-digital converter (ADC), whose output is a string of numbers

$$SW = |W_k - W_{k-1}|$$

Need at least one in order to know we don't have either numbers (real part) numbers (imaginary part)

This time-domain signal:

$\Delta t$: dwell time
$N$: number of complex data points acquired

The discrete sampling implies that the integral of the Fourier transformation has to be replaced by a discrete sum

$$\int_{-T}^{T} \rightarrow \sum_{m=-\infty}^{N-1}$$

The acquisition time determines the spectral resolution $\Delta \nu$ of the spectrum:

$$\Delta \nu \cdot AT = 1$$

Similarly, the dwell time $\Delta t$ determines the spectral width (SW) of the spectrum:

$$SW \cdot \Delta t = 1$$

Nyquist Theorem

$SW$ defined

\begin{align*}
\int e^{i \omega (t-nT)} dt & \neq 0 \\
\int e^{i (\omega_k - \omega_l) t} dt & \neq 0
\end{align*}
The detected signal

\[ S(t) = S_x(t) + i S_y(t) \quad t = m \cdot \Delta t \]

The resulting spectrum

\[ F(\nu) = A(\nu) + i D(\nu) \quad \nu = k \cdot \Delta \nu \]

\[ F_k = A_k + i D_k \quad k = \frac{N}{2}, \ldots, N-1 \]

The FT:

\[ F(\omega) = \frac{1}{\sqrt{\Delta T}} \int_{-T}^{T} S(t) e^{-i \omega t} dt \]

\[ T = N \Delta t \quad dt \rightarrow \Delta T \quad \omega = 2 \pi \nu \quad T = 0 \]

Discrete version

\[ F_k = \frac{1}{N} \sum_{m=0}^{N-1} S_m e^{-i 2 \pi k m / N} \]

This is very convenient to program => computers can calculate \( F_k \) very fast if

\( N = 2^m \) using an algorithm called the fast Fourier transform (FFT)

\[ F_k = \text{complex} \quad \text{or} \quad S(t) = \int F(\omega) e^{i \omega t} d\omega \]

\[ \text{Probability} \quad \text{Spins are resonating at } \nu \]

\[ \text{SW} = \Delta \nu \times (NP-1) \]

\[ \text{AT} = \Delta t \times (NP-1) \]
II.5 EFFECTS OF RELAXATION: THE BLOCH EQUATIONS

So far we have that:

i) Before excitation \( \vec{M} = M_0 \hat{z} \)

ii) After excitation \( \vec{M}(t) = M_0 (\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}) \)

However, if one waits long enough, the magnetization is again parallel to \( \hat{z} \).
There is a mechanism that takes the system back from ii) to i): Relaxation.

Two relaxation times determine how this process takes place:

A "transverse" relaxation time \( T_2 \): Takes \( M_y, M_x \to 0 \)

A "longitudinal" relaxation time \( T_1 \): Takes \( M_z \to M_0 \)
The longitudinal growth and the transversal decay are usually exponential.

\[ T_1 \text{ or longitudinal relaxation} \]
\[ T_2 \text{ or transverse relaxation} \]

\[ M_z(t) = M_o (1 - e^{-t/T_1}) \]
\[ M_{xy}(t) = M_o e^{-t/T_2} \]

These processes can be included into the equation of motion of \( M \) to give the Bloch Equations:

\[ \begin{align*}
\dot{M}_x &= -\Delta \omega M_y - \frac{M_x}{T_2} \\
\dot{M}_y &= \Delta \omega M_x - \frac{M_y}{T_2} \\
\dot{M}_z &= \frac{M_o - M_z}{T_1}
\end{align*} \]

These 2 expressions can be rewritten as

\[ \dot{M}_x = i \Delta \omega M_x - \frac{M_x}{T_2} \]

where

\[ M_+ = M_x + i M_y \]

\[ M_- = e^{i \Delta \omega} M_+ \]

\[ M(t) = M(0) e^{i \Delta \omega t} - \frac{i M_x}{\Delta \omega} e^{-t/T_2} \]
II.6 NMR LINESHAPES AND THE PHASE OF SPECTRAL PEAKS

Due to $T_2$ effects the actual time-domain signal looks like:

$$S(t) = S_0 e^{i\omega t} e^{-t/T_2} e^{i\phi}$$

The detected signal is referred to as FID

FID: Free Induction Decay

For a single $\omega_0$:

$$S(t) = S_0 e^{i\omega_0 t} e^{-t/T_2}$$

Complex FT:

$$F(\omega) = \int_{-\infty}^{\infty} S(t) e^{-i\omega t} dt$$

$$B = \text{complex}$$

At $t=0$

$$A(\omega): \text{absorptive line shape}$$

$$D(\omega): \text{dispersive line shape}$$

Lorentzian Line Shape

$$A(\omega) = \frac{1 \cdot S_0 \cdot T_2}{1 + (\omega - \Delta \omega)^2 T_2^2}$$

$$D(\omega) = \frac{T_2^2 (\omega - \Delta \omega)}{1 + (\omega - \Delta \omega)^2 T_2^2}$$

$T_2$ takes time to decay

$T_2$ takes time to cool down
Recall that the signal from the DBM was actually

\[ S(t) = S_0 e^{iωt} e^{iφ} \]

\[ Δω \text{: offset} \quad φ = φ_{r,x} - φ_{T,x} \]

Moreover, if one takes into account that:

i) the magnetization starts to precess as soon as it departs from the z-axis

ii) after we turn off the rf-pulse, we have to wait some time ("dead time") until we can start to digitize the signal

Rec 10 MV

Trns 10 V
\[ S(t) = S_0 \, e^{i \omega t + i \varphi} \]

\[ \Delta = \varphi + \Delta \omega \cdot t \]

\[ \mathcal{F}(w) = \left[ A(w) \cdot \cos \varphi - D(w) \sin \varphi \right] + i \left[ D(w) \cos \varphi + A(w) \sin \varphi \right] \]

\[ A(w) \quad \text{purely absorptive} \]

\[ D(w) \quad \text{purely dispersive} \]

**Phase correction**

\[ R' = \cos \varphi \cdot R + \sin \varphi \cdot \mathcal{R} \]

\[ \mathcal{R}' = \cos \varphi \cdot \mathcal{R} - \sin \varphi \cdot \mathcal{R} \]

\[ \omega t = + \quad \omega t = - \]

\[ \mathcal{F}(w) = \left[ A(w) + i \cdot D(w) \right] \cdot e^{i \theta} = R + i \cdot \mathcal{R} \]

**Note:** The fact that we have dispersive components is a consequence of the impossibility of knowing the signal for \( t < 0 \) (causality of NMR).
Informal Proof: We know that given a signal $S$

$$S(t > 0) = S_0 e^{i\omega t}$$

$$\Rightarrow \mathcal{F}_{\text{old}}(w) = \int_0^\infty S(t) e^{-i\omega t} dt = A(w) + iD(w)$$

If the signal goes from $-\infty$ to $\infty$, the new spectrum

$$\mathcal{F}_{\text{new}}(w) = \int_{-\infty}^\infty S(t) e^{-i\omega t} dt$$

$$= \int_0^\infty S(t) e^{-i\omega t} dt + \int_{-\infty}^0 S(t) e^{-i\omega t} dt$$

$$= \int_0^\infty S(t) e^{-i\omega t} dt + \int_0^\infty S(-t) e^{i\omega t} dt$$

$$= 2A(w)$$

Q.E.D.
II.7 SIGNAL-TO-NOISE (S/N) RATIO IN PULSED NMR

NMR relies on $M_0 = $ excess of $\alpha$ spins ($\uparrow$) over $\beta$ spins ($\downarrow$). This is a very small fraction, and therefore a normal FID looks like $T_2 = 1$ sec

Noise is the main drawback of NMR spectroscopy. NMR spectroscopy bypasses this problem by signal averaging

The acquisition computer adds the data digitized in each scan point by point

$\sum : \ldots$
The intensity of the **signal** adds up coherently \( \Rightarrow S \propto \# \text{ of scans} \)

The intensity of the **noise** adds up randomly \( \Rightarrow N \propto \sqrt{\# \text{ of scans}} \)

\[ \Rightarrow \quad \text{S/N} \propto \sqrt{\# \text{ of scans}} \]

Since the detected signal decays as \( e^{-t/T_2} \Rightarrow \) there is no point in using \( AT \geq 3T_2 - 4T_2 \)

The spin system starts to relax right after the rf pulse. If the excitation pulse is \( \pi/2 \) no \( M_0 \) is left parallel to \( z \); if the excitation pulse is \( \pi/4 \) we get only 70\% of the maximum possible signal, but 70\% of \( M_0 \) is still parallel to \( z \Rightarrow \pi/2 \) pulses do not necessarily afford the best S/N.

In general, if \( T_1 = T_2 \) (most liquids) \( \Rightarrow \) the optimum excitation angle \( \beta_{\text{opt}} \) is given by

\[ \cos (\beta_{\text{opt}}) = e^{-AT/T_1} \; \text{; with delay time} = 0 \]

Whereas the noise is constant throughout the FID the signal decays exponentially with time. Thus, it is possible to increase S/N by giving more "weight" to the beginning of the FID than to the end. This is achieved by multiplying the FID by a window function.
The optimal window function matches the shape of the FID envelope.

Figure 2.18 Application of the matched filter improves sensitivity.

Figure 2.20 Resolution enhancement by the Lorentz-Gauss transformation. The lower traces show the natural FID and its transform, while in the upper trace the early part of the decay has been cancelled by the window function (which still ensures apodisation). Such strong manipulation as this one only be applied to data with a very high signal-to-noise ratio.
II.8 RESOLUTION CONSIDERATIONS

Although the head of the FID carries most of the signal, the tail carries the resolution: the longer the FID, the sharper the peaks.

Resolution can be improved at the expense of S/N by using an exponential weighting with LB < 0. Lorentz-Gauss transformation may improve both resolution and sensitivity.

Digital resolution can be improved by zero filling.

Figure 2.14 Zero-filling the time domain data interpolates extra points into the frequency spectrum, improving its appearance.
In liquid NMR, the main factor conspiring against resolution is field inhomogeneity. For instance:

Due to the field inhomogeneity the decay of the FID is usually given by a $T_2^* < T_2$.

The homogeneity required from an NMR magnet is extremely high: Over the sample volume (= 1 cm diameter x 1 cm height cylinder), the linewidth at = 500 MHz should be $≈ 0.05$ Hz $⇒$ Inhomogeneity $= \frac{5 \times 10^{-2}}{5 \times 10^8} = 1$ part in $10^{10}$.

The effects of field imperfections:

```plaintext
$\gamma \chi M$
```

Correct

$xy, x^2, y^2$

(2nd ORDER SIDEBAND)

$z$

(1st ORDER SIDEBAND)
To achieve such resolution a "perfect" magnet and completely non-magnetic materials are not enough, one has to resort to shimming: the application of small magnetic field gradients that compensate the inhomogeneities of the main coil.

**Supercon shims:** usually 4 gradients tuned only upon installation.

There are two sets of shim coils.

**Room temperature shims:** usually 18-24 gradients, most of them adjusted upon installing the observation probehead containing the sample; some of them need to be adjusted everyday.
Even shims are not enough; the highest homogeneity is achieved by spinning the sample at \( \approx 20-40 \) Hz.

\[ \text{Spinning averages out inhomogeneities in the } x-y \text{ plane } \Rightarrow \text{attention is focussed on the } z-axis \text{ shims.} \]

Although shimming and spinning can eliminate instantaneous inhomogeneities, sharp lines also require the elimination of long term drift:

\[ t_1 \quad t_2 \quad t_3 \quad \sum \]

\[ \omega_0 \quad \omega_1 \quad \omega_2 \quad \omega_3 \]

This is achieved using a deuterium lock: a system which keeps the magnetic field locked (on-resonance) on a \(^2\text{H} \) NMR signal.
At the \(^2\text{H} \) NMR frequency

\[ ^2\text{H} \text{ NMR FID} \]

The deuterium lock system keeps \( B_0 \) constant by monitoring \( \int S_y(t) \, dt \)

\[ \Delta \omega > 0 \]

\[ \text{on resonance} \]

\[ \Delta \omega < 0 \]

Linear region \( \Rightarrow \int S_y(t) \, dt \propto \text{error signal} \)
II.9 NON-QUADRATURE DETECTION AND QUADRATURE GHOSTS

We saw that given a signal

\[ S(t) = S_0 e^{i \omega t} e^{-t/T_2} = S_0 e^{-t/T_2} \left[ \cos(\omega t) + i \sin(\omega t) \right] \]

\[ F(\omega) = A(\Delta \omega) + i D(\Delta \omega) \]

Using some basic trigonometric relationships

\[ e^{i \varphi} = \cos \varphi + i \sin \varphi \]
\[ e^{-i \varphi} = \cos \varphi - i \sin \varphi \]

\[ \cos \varphi = \frac{(e^{i \varphi} + e^{-i \varphi})}{2} \]
\[ \sin \varphi = -i \frac{(e^{i \varphi} - e^{-i \varphi})}{2} \]

One can get further insight into how the \( A(\Delta \omega) + i D(\Delta \omega) \) line shape is obtained. Consider these cases:

- **Time domain**
  - R : \( \cos \)
  - I : 0
  - R : 0
  - I : \( \sin \)

\[ \Rightarrow \cos + i \sin \]

\[ F \text{ of cos} \rightarrow \text{Mirror image} \]

\[ F \text{ can't make identical back} \]

\[ \text{It can make identical form} \]

\[ \text{But can't make identical back} \]
Old systems used non-quadrature detection ($R = \cos; \, I = 0$). In these cases one always has to work off-resonance and throw away half the points, but even though peak folding can be avoided noise folding cannot.

Even in quadrature detection it is impossible to make the gain of both channels identical; this artifact appears as a quadrature ghost.

Figure 4.32: This alternative placement of the detector reference circumvents the problem of negative frequencies, but with single-phase detection extra noise (to the right of the reference line) will be combined into the spectrum.
II.10 PHASE CYCLING

An additional potential artifact that affects pulsed NMR experiments is a small DC offset added to the signal:

\[ S_x(t) = S_0 \cos(\Delta \omega t + \Delta x) \]
\[ S_y(t) = S_0 \sin(\Delta \omega t + \Delta y) \]

If known, subtraction of the average baseline noise from the FID ("baseline correction") can eliminate the peak at zero. A more general procedure which can eliminate both baseline and quadrature problems is phase cycling.

Recall that our signal was actually

\[ S(t) = S_0 e^{i \Delta \omega t} e^{i \varphi} \quad \varphi = \varphi_{R_x} - \varphi_{T_x} \]

Consider four signals arising from 4 independent experiments:

<table>
<thead>
<tr>
<th>Phase of pulse</th>
<th>Phase factor</th>
<th>( \cos(\Delta \omega t + \varphi) )</th>
<th>(-\sin(\Delta \omega t + \varphi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_{R_x} )</td>
<td>( e^{i \varphi} )</td>
<td>( \cos(\omega t + \varphi) )</td>
<td>( -\sin(\omega t + \varphi) )</td>
</tr>
<tr>
<td>( \varphi_{T_x} + \frac{\pi}{2} )</td>
<td>( e^{i \pi / 2} = i e^{i \varphi} )</td>
<td>( -\sin(\omega t + \varphi) )</td>
<td>( -\cos(\omega t + \varphi) )</td>
</tr>
<tr>
<td>( \varphi_{T_x} + \pi )</td>
<td>( e^{i \pi} = -e^{i \varphi} )</td>
<td>( -\cos(\omega t + \varphi) )</td>
<td>( \sin(\omega t + \varphi) )</td>
</tr>
<tr>
<td>( \varphi_{T_x} + \frac{3\pi}{2} )</td>
<td>( e^{i 3\pi / 2} = -i e^{i \varphi} )</td>
<td>( \sin(\omega t + \varphi) )</td>
<td>( \cos(\omega t + \varphi) )</td>
</tr>
</tbody>
</table>

Thesignal:

\( \sin(\Delta \omega t + \varphi) \)
\( \cos(\Delta \omega t + \varphi) \)
\(-\sin(\Delta \omega t + \varphi) \)
\(-\cos(\Delta \omega t + \varphi) \)

Add them up while changing Rx phase.
The signals that can be expected from these experiments:

If these signals are added with \( \Phi_{\text{RX}} = \text{constant} \), the result is 0, but if the receiver phase is incremented by \( \pi/2 \) in each experiment, the signals add up coherently:

\[
\Phi_{\text{RX}} - \Phi_{\text{TX}} = \Phi = \text{constant}
\]

However, quadrature ghosts and baseline offsets disappear upon phase cycling.

**MAIN PEAK**

- Change Rx Phase: 1. Remove Rx, cable, ADC 1, ADC 2.
- Cycle: 1.
  - 1st Rx: \( \Phi \rightarrow 1 \)
  - 2nd Rx: \( \Phi \rightarrow 2 \)
  - 3rd Rx: \( \Phi \rightarrow (-1)^2 \)

**IMAGE PEAK**

- Cycle of phases x 32

**IMAGE WITH CYCLOPS**

- Buffer 1 & 2

**HERTZ**

- 250 200 150 100 50 0 -50 -100 -150 -200 -250
Changing $\varphi_{T_x}$ requires 90° phase shifts of the $T_x$ rf; changing $\varphi_{R_x}$ however can be done by software:

![Diagram of NMR RX system with ADCs and buffers]

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>ADC1</th>
<th>ADC2</th>
<th>Buffer1(*)</th>
<th>Buffer2(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>cos</td>
<td>sin</td>
<td>+ADC1</td>
<td>+ADC2</td>
</tr>
<tr>
<td>2</td>
<td>-sin</td>
<td>cos</td>
<td>+ADC2</td>
<td>-ADC1</td>
</tr>
<tr>
<td>3</td>
<td>-cos</td>
<td>-sin</td>
<td>-ADC1</td>
<td>-ADC2</td>
</tr>
<tr>
<td>4</td>
<td>sin</td>
<td>-cos</td>
<td>-ADC2</td>
<td>+ADC1</td>
</tr>
</tbody>
</table>

(*) + : add; - : subtract

Homewo: Quad phases if dc off are elim

ADC1: DC off 1
ADC2: DC off 2
### II.11 REVIEW OF ELECTRONICS

When dealing with DC currents or voltages, circuits are described by their resistance $R$:

$$
\frac{\text{Vin}}{\text{circuit}} \xrightarrow{\text{Vout}} \frac{\text{Vin} - \text{Vout}}{R} = R \cdot I
$$

In AC (Audio $\leftrightarrow$ kHz, rf $\leftrightarrow$ MHz or microwave $\leftrightarrow$ GHz) circuits, voltages and currents are time-dependent functions $A \cdot \cos (\omega t + \phi)$, usually described by complex functions:

$$
V = V_0 e^{i(\omega t + \phi)} \quad ; \quad I = I_0 e^{i(\omega t + \phi)}
$$

Ohm's law is still valid in this context, but instead of resistances we now talk about impedances ($Z$):

$$
\frac{\text{Vin}(t)}{\text{circuit}} \xrightarrow{\text{Vout}(t)} \frac{\text{Vin}(t) - \text{Vout}(t)}{Z} = Z \cdot I(t)
$$

The three basic linear components of a circuit are:

- **Resistor**: $Z_R = R$ (Real)  
  - R: resistance (Ω)
- **Capacitor**: $Z_C = -j/\omega C$ (Complex)  
  - C: capacitance (μF, pF)
- **Inductor**: $Z_L = j\omega L$ (Complex)  
  - L: inductance (μH)

The impedance of a complex circuit can be expressed in terms of the impedance of its components according to:

- **Series Circuit**: $Z_{eq} = Z_1 + Z_2$
- **Parallel Circuit**: $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

At high frequencies, series components dominate and parallel components can be ignored. At low frequencies, parallel components dominate and series components can be ignored.
An important non-linear element is the diode. A diode behaves like a resistor when a large enough voltage is applied, but does not let current flow when voltage is applied.

Many applications require a "crossed-diodes" configuration.

\[
\begin{cases}
    I_f \quad V > V_{\text{thresh}} \\
    I_f \quad V < V_{\text{thresh}}
\end{cases}
\]
**II.12 NMR PROBES**

A basic NMR experiment requires:

- **rf Tx**
- **Sample**
- **Bo**
- **rf Rx**
- **NMR probehead (probe)**
- **Irradiating the sample efficiently with a B0 oscillating at ω0**
- **Detecting the signal from the spins precessing at ω0**

In an NMR experiment we need a probehead capable of:

- Most effective transfer from Tx to probe

Need $Z_{probe} = Z_{rx}$ (Impedance must be same as Larmor field.)

If the same coil is used for irradiation and detection these 2 conditions are equivalent: a system capable of delivering rf efficiently, will also detect small signals efficiently.

The electronic properties of the probe are described by the probe's impedance $Z_{probe}$. To achieve maximum efficiency in the

- **rf transmission**: $Z = Z_{probe}$ at $ω = ω_0$
- **rf detection**: $Z = Z_{probe}$ at $ω = ω_0$

For maximum flow want $R_1$ of $L_1 = R_2$ of $C_2$
Similar problems arise in other situations involving rf, where it was realized that things could become simpler if everyone uses devices with the same impedance. This was chosen as:

\[ Z = 50 \, \Omega \]  
\[ Z_{\text{everything}} = 50 \, \Omega \]  
\[ \text{at our normal} \]

For all compatibility, eg) amp DBN of 50D.

To tune a probe means taking the impedance of the irradiation/detection coil at \( \omega_0 \) to 50 \( \Omega \). This can be achieved using 2 additional capacitors in the following configuration:

\[ L, R: \text{inductance and parasitic resistance of sample coil} \]

If for such a circuit

\[ C_1 = \frac{\sqrt{1/50}}{\omega_0^2 L} \]
\[ C_2 = 1 - \frac{\sqrt{1/50}}{\omega_0^2 L} \]

Then,

\[ Z_{\text{probe}} = 50 \, \Omega \]

Values of 2 capac \( \rightarrow \) 50 \( \Omega \)

\[ L \rightarrow \text{match effects height} \]
\[ \text{match channel height} \]

Changes shown: match channel \( \pm \).
II.13 DUPLEXING

We have the system connected in such a way that

\[ \text{Tx} \rightarrow \text{Rx} \]

\( \approx 100 \text{V} \quad \approx 10 \text{mV} \)

Delivers high rf powers
Detects very small rf signals

If the Rx is not isolated from the Tx, the rf burst will burn it.

There are 2 ways of isolating (duplexing) the rf on such a system:

a) Use 2 orthogonal coils, each with its own set of capacitors:

\[ \mathbf{B}_1 \rightarrow \mathbf{B}_0 \]

\( \mathbf{z} \)

\( \mathbf{r}_x \)

Advantage: By construction, the Tx and Rx don't see each other.
Disadvantages: i) The Tx coil is larger \( \Rightarrow \) less efficient (coil for flux sample)
ii) Only works for Helmholtz coils (= 1/3-1/2 as efficient as solenoid coils)

b) Use an active or passive duplexing system

Too many diodes & transistors add noise.
that looks like

\[ T_x \rightarrow \text{Probe} \] when the \( T_x \) pulse is sent

\[ \text{Probe} \rightarrow R_x \] when the spin signal is detected

This can be achieved using crossed diodes and a quarter-wave (\( \lambda/4 \)) coaxial cable.

A coax:

grounded \( \lambda/4 \):

- \( V = 0 \) at \( V_{\text{max}} \)
- Standing wave

open \( \lambda/4 \):

- Virtual ground
- Standing wave

A simple scheme of an rf duplexer:

\[ T_x \rightarrow \text{Probe} \rightarrow \text{Duplexer} \rightarrow R_x \]

\( \lambda/4 \): Pulse builds Standing wave

\( \lambda/4 \): Goes to \( R_x \)
II.14 BROADBAND IRRADIATION

In general, an NMR experiment is done off-resonance. Under these conditions it is difficult to apply accurate $\pi/2$ or $\pi$ pulses:

$\Delta \omega = 0$

circle on the $x$-$y$ plane

$\Delta \omega \neq 0$

eclipse: crosses the $x$-$y$ plane but not the $y$-axis

does not touch the $y$-axis

cant give $\pi$ pulse here

Even if $\Delta \omega = 0$, spatial inhomogeneities of the rf field produce problems

$B_1$ inhomogeneity much larger than $B_0$

$1 \times 10^9$ for $B_0$

$B_1$ inhomogeneity

$10$ mm can't give $\pi$ pulses

Sometimes see

Because of the curve then
It is possible to minimize these imperfections by replacing $\pi/2$ or $\pi$ pulses by composite pulses: sequences of rf irradiation whose net effect are those of an ideal $\pi/2$ or $\pi$ pulse.

E.g.: if we have inhomogeneity in $B_1$, we can replace a $\pi_x$ pulse by

$$\left\{ (T/2)_x, (\pi)_y, (T/2)_x \right\}$$

not really exact remember they are for new

$$\begin{array}{c}
M \xrightarrow{(\pi/2)_x} M, B_1 \xrightarrow{m_1, B_1} M, (\pi)_y, M \xrightarrow{(\pi/2)_x} M \end{array}$$

does $\alpha_0$ about self

For inhomogeneous $B_1$:

For $90^\circ$ pulse

$$\begin{array}{c}
90^\circ \xrightarrow{\text{pulse}} (\pi/2) \equiv \left\{ (\pi/4)_y, (\pi/2)_x, (\pi/2)_y, (\pi/4)_x \right\}
\end{array}$$

For offset $\Delta\omega$:

For $180^\circ$ w/ some offset

$$\begin{array}{c}
180^\circ \xrightarrow{\text{offset}} \pi \equiv \left\{ (\pi/2)_x, (4\pi/3)_y, (\pi/2)_x \right\}
\end{array}$$
II.15 SELECTIVE IRRADIATION

The excitation spectrum of an rf pulse is approximately given by its Fourier spectrum:

\[
\text{Sinc function } \left( \frac{\sin x}{x} \right) \text{ centered at } \nu_0.
\]

If pulse is long \( \Rightarrow \) get small excit B. W.

Sometimes, one wants to restrict excitation to a narrow region of the frequency domain. There are several ways of achieving this:

i) Using long rectangular pulses:

ii) Using shaped pulses:

Gaussian (relatively easy)

Sinc (more difficult)
iii) Using a **DANTE** (Delays Alternating with Nutations for Tailored Excitation) sequence: train of short rectangular pulses of angle $\alpha$

\[ \frac{\pi}{2} \quad \frac{\pi}{2} \quad \alpha \quad \frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{\pi}{2} \]

$N$ pulses; $N \approx T$

chosen so that $N \cdot \alpha = \frac{\pi}{2}$ for $\frac{\pi}{2}$ pulse

\[ \text{Excitation profile} \]

iv) Using composite pulses; e.g.

\[ (\frac{\pi}{2})_x \cdot \text{e}^{(\frac{\pi}{2})_x} \cdot \text{e}^{(\frac{3\pi}{2})_x} \cdot \text{e}^{(\frac{3\pi}{2})_x} \cdot \text{e}^{(-\frac{\pi}{2})_x} \cdot \frac{\pi}{2} \]

Vary $x$ then vary $\alpha$ for various effects

Why use DANE (Selective excitation) e.g. for $^{1}H_2O$
II.16 BLOCK DIAGRAM OF A BASIC NMR SPECTROMETER

- rf synthesizer
- computer
- pulse programmer
- 90° splitter
- 0° 90°
- dbm
- ref freq
- quad act
- rf receiver
- bat
- beam amplifier
- detector
- audio amplifiers
- low -> higher
- power rf amp
- high pin
- output of db signals
- lab freq switches on
- off
- magnet
- sample
- nmr coil
II.17 PROBLEMS

1) The sensitivity problem in NMR: Using the expressions for $M_0 = M_0 (\gamma B_1 T)$ and for the field produced by a magnetic moment, make an estimate of the voltage that will be induced during an NMR experiment on a 1-turn coil for the following arrangement. Assume a $\frac{\pi}{2}$ pulse.

![Diagram of a coil with a sample and field lines]

$B_0 = 7 \, \text{T}$

Sample with $10^{20}$ protons at $T = 300 \, ^\circ \text{K}$

$\omega_0 = 300 \, \text{MHz}$

How will this voltage change by adding a second turn to the coil?

2) At what magnetic field will the $^{13}\text{C}$ NMR signal arising from 1mL CHCl$_3$ be as intense as the $^1\text{H}$ NMR signal arising from the same sample at 2.35 T ($\gamma_{^1\text{H}}/\gamma_{^{13}\text{C}} = 4$)?

3) (a) Given 2 input signals: $s_1 = \cos (\omega_1 t + \phi_1)$, $s_2 = \cos (\omega_2 t + \phi_2)$

   Calculate the phase and frequencies of the output signal coming from a DBM.

   (b) How can a DBM be used as a:
      i) phase-sensitive detector
      ii) frequency doubler
      iii) 180° phase shifter
      iv) rf attenuator
      v) rf gate

4) Adiabatic Inversion: Show that an alternative to $\pi$ pulses for inverting a magnetization vector is to sweep slowly enough a $B_1$ field from well-above to well-below exact resonance.
5) The time- vs the frequency-domain: Calculate the Fourier transforms of the following functions:

<table>
<thead>
<tr>
<th>Time-domain</th>
<th>Frequency-domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>rf pulse</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Gaussian pulse</td>
<td></td>
</tr>
<tr>
<td>$\sin \pi t$</td>
<td></td>
</tr>
<tr>
<td>$\delta$ pulse</td>
<td></td>
</tr>
<tr>
<td>$\int \delta(t) dt = 1$</td>
<td></td>
</tr>
</tbody>
</table>

- Exponential decay: $e^{-at}$, $a > 0$
- Gaussian decay: $e^{-at^2}$
6) What are the mathematical expressions of the inverse Fourier transform in continuous and in discrete sampling cases.

7) **Peak folding:** Nyquist theorem states that given a dwell time $\Delta t$, we can only characterize peaks within the following range:

$$f_s = \frac{1}{\Delta t}$$

i) How many points would be sampling a resonance precessing at $v = SW/2$?

ii) Use the following picture to estimate where a resonance precessing at a rate $v = SW$ would appear.

![Image of peak folding diagram]

iii) Where would a $v = -SW$ resonance appear? On which region of the spectrum would a resonance precessing between $v = sw/2$ and $v = sw$ appear?

Cases ii) and iii) are examples of peak folding: if $\Delta t$ is not chosen correctly, peaks appear where they shouldn't.

8) What dwell time should be used and how many points should be acquired to obtain a spectrum characterized by a spectral window of 10 kHz and a digital resolution of 0.5 Hz? (recall that usually the # of points = $2^m$)

9) Calculate the first 8 points of an NMR signal arising from a site whose
resonance offset is 1000 Hz and line width = 1 Hz; assume a dwell time of 0.5 ms

10) How does the total length of a magnetization vector change with time after a π/2 pulse in the case of T₁ = T₂?

11) Give the expression for the Bloch equations in the presence of a B₁ field placed at an arbitrary orientation along the x-y plane.

12) NMR line shapes: Calculate the FT of \( S(t) = e^{i \Delta \omega t} e^{-t/T₂} \)

13) The justification of phasing: Find the expression for the full-width at half-height of:
   i) a lorentzian line shape A(ω)
   ii) a magnitud representation of the signal = \( \sqrt{A(\omega)^2 + D(\omega)^2} \)

On the basis of the above analysis, justify the use of phasing.

14) The sequence of events involved in a 1-pulse NMR experiment:

   \[ \text{rf pulse} \quad \text{dead time} \quad \text{turn on } R_x \quad \text{acquire FID} \]

   Typical event durations: 10µs 50µs 100µs
If an experiment carried on-resonance affords a peak that requires a phase-correction of $\phi = 45^\circ$ to become purely absorptive, what phase correction will be needed if the transmitter is moved

i) +5 kHz off-resonance

ii) -5 kHz off-resonance

Use the typical time delays listed above.

15) Echo experiments: An NMR echo affords a time-domain FID of the type

Schematize the real and imaginary parts of the spectrum obtained by $\mathcal{F}$-ing this signal. What phase correction has to be applied after $\mathcal{T}$ to get a purely absorptive line shape if the constant phase correction is zero, and a total of 1024 points were acquired using a dwell time of 500 $\mu$s ($\mu$s = $10^{-6}$ s). Could a purely absorptive line shape be obtained without phasing? If so, how?

16) If 1mL of CHCl$_3$ gives a $^{13}$C NMR spectrum with a S/N = 10 in 1 scan (total $\Delta T = 1$ sec), how long will it take to reach a S/N = 100 with a 0.2 mL of sample under identical experimental conditions?

17) Ernst's angle: Demonstrate the validity of the equation defining the optimum excitation conditions ($\beta_{\text{opt}}$ = Ernst angle). See page 55.

18) Truncation: The number of acquisition points used in an NMR experiment has to be chosen large enough to let the signal decay to ca. the level of the noise. Calculate the line shape obtained if only one quarter of this number of points is
acquired. Hint: The result is the convolution of the normal line shape with the FT of a step function:

from normal signal  from truncated signal

How can the wiggles originating by truncating the signal be eliminated without increasing the number of acquired points? At what cost?

19) Why isn't it convenient to spin the sample faster than ca. 50 Hz when recording a high-resolution NMR spectrum?

20) Calculate the signals arising from an FID whose real and imaginary components are:
   i) $R \rightarrow \sin(\omega t)$; $I \rightarrow 0$
   ii) $R \rightarrow 0$, $I \rightarrow \cos(\omega t)$

21) Coherence selection: from a quantum mechanical point of view, an NMR experiment detecting a signal $S(t) = Tr(\rho \, I_+) = \text{const} \cdot e^{-i \omega t}$ is said to be detecting the -1 coherence (the $I_-$ component of $\rho$). How is it necessary to rearrange the NMR data so as to detect the +1 coherence (i.e., to obtain a peak at $-\Delta \omega$ from the same spin system).

22) Dynamic Range: Final digitization of an NMR signal involves the following set up:
An ADC is characterized by the largest voltage \( V_{\text{max}} \) that it can represent and by the largest digit \( (N_{\text{max}}/2) \) assigned to this voltage. A typical value for \( V_{\text{max}} = 5V; \) \( N_{\text{max}} \) is given by the \# of bits in the digitizer (usually 12 or 16), according to \( N_{\text{max}} = 2^{\# \text{ of bits}}. \) This number determines the dynamic range of an NMR spectrometer; i.e., what is the ratio between the strongest signal to the smallest significant feature that can be detected in an experiment. Digitized signals from successive NMR experiments are then added into a computer word whose maximum value \( W_{\text{max}}/2 \) is given by the \# of bits in the word (usually 20 or 24: \( W_{\text{max}} = 2^{20} \) or \( 2^{24} \)).

i) What is the dynamic range of a 12 bit digitizer? And of a 16 bit digitizer?

ii) What should the voltage of the acquired signal be set to in order to observe a weak NMR resonance under optimal conditions?

iii) Given the acquisition conditions of item ii), calculate how many scans can be acquired using a 16-bit digitizer before overflowing (i.e., reaching the maximum value) of a 24-bit word for a system where (a) the S/N = \infty; (b) the S/N = 0. (Remember that noise adds up randomly)

23) Demonstrate that an imbalance \( \Delta G \) in the gain of the real and imaginary audio channels gives origin to a quadrature ghost. Calculate the intensity of the ghost as a function of \( \Delta \omega, \Delta G \).

24) Fill out the following table:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \phi_{\text{rx}} )</th>
<th>( R_{\text{signal}} )</th>
<th>( I_{\text{signal}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>( \cos \Delta \omega t )</td>
<td>( \sin \Delta \omega t )</td>
</tr>
<tr>
<td></td>
<td>( \phi/2 )</td>
<td>(-\sin(\Delta \omega t))</td>
<td>( \cos(\Delta \omega t) )</td>
</tr>
<tr>
<td></td>
<td>( \pi/4 )</td>
<td>(-\cos(\Delta \omega t))</td>
<td>( -\sin(\Delta \omega t) )</td>
</tr>
<tr>
<td></td>
<td>( \pi/2 )</td>
<td>( \sin(\Delta \omega t))</td>
<td>( -\cos(\Delta \omega t) )</td>
</tr>
<tr>
<td></td>
<td>( \pi )</td>
<td>(-\cos(\Delta \omega t))</td>
<td>( \sin(\Delta \omega t) )</td>
</tr>
</tbody>
</table>
25) Given \( n \) phase-shifted experiments characterized by transmitter rf phases
\[
\varphi_x = \varphi_0^x + 2\pi (i-1) / N ; \quad i = 1, 2, \ldots, N ; \quad \varphi_0^x = \text{any} ;
\]
i) Calculate the coefficients \( c_1^i, \ldots, c_4^i \) of the linear combinations
\[
c_1^i \cdot \text{ADC 1} + c_2^i \cdot \text{ADC 2} \rightarrow \text{memory buffer 1},
\]
\[
c_3^i \cdot \text{ADC 1} + c_4^i \cdot \text{ADC 2} \rightarrow \text{memory buffer 2},
\]
that are needed to add the signals of the different experiments coherently.
ii) Find the minimum \( N \) that will eliminate DC offsets. Will this phase cycle also cancel quadrature ghosts?
iii) Find the minimum \( N \) that will eliminate quadrature ghosts. Will this phase cycle also cancel DC offsets?

26) Calculate the equivalence impedance of:
   i) 2 resistors in parallel/series
   ii) 2 capacitors in parallel/series
   iii) 2 coils in parallel/series

27) Calculate the impedance \( Z(\omega) \) of
   i) A series R-L-C circuit

\[
V = V_0 e^{i \omega t}
\]
\[
I = Z e^{i \omega t}
\]
ii) A parallel R-L-C circuit

Graph the behavior of $Z_{eq}$ as a function of $\omega$ (i.e., the frequency response of the system). Note what happens at the resonance condition $\omega = (LC)^{-1/2}$.

28) In DC systems, the power delivered to a load $R$ is given by

$$P = \frac{V^2}{R} = I^2 R$$

In AC systems, these equations become

$$P = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 \cdot R,$$

Where

- $V_{\text{pp}} = 2 \cdot V$, $V_{\text{rms}} = V / \sqrt{2}$
- $I_{\text{pp}} = 2 \cdot I$, $I_{\text{rms}} = I / \sqrt{2}$

A measure of relative power is the decibel or db:

\[
\text{(db): relative power} = 10 \cdot \log \frac{P_{\text{in}}}{P_{\text{out}}} = 20 \cdot \log \frac{V_{\text{in}}}{V_{\text{out}}}
\]

i) What \( V_{\text{pp}} \) corresponds to 100 mw sent to a 50 \( \Omega \) load?

ii) What's the relationship between the measured \( V_{\text{pp}} \) and the incoming power \( P \) in the following setup:

\[
\text{30 db attenuator} \rightarrow V_{\text{pp}}(P) = ?
\]

\[
\text{40 db attenuator} \rightarrow V_{\text{pp}}(P) = ?
\]

29) **Maximum power transfer theorem:** Consider the following setup, analogous to the one characterizing the transmission of rf to the probe or the reception of rf from it:

Demonstrate that the condition of maximum power transfer from the source to the load is given by

\[ Z_S = Z_L \]

Hint: Power \( P_L = I \cdot V_L \); \( I = V_S/Z_{eq} \)

\[ V = \frac{V_S}{Z_{eq}} \]

\[ \rho = \frac{V}{\sqrt{R}} = \frac{P}{\sqrt{R}} \]

\[ I^2 = \frac{V^2}{(Z_S + Z_L)^2} \]
30) i) A λ/4 cable can be measured using the following setup:

What should V ideally be when the coax is a λ/4 cable for the proper frequency ω?

ii) An NMR probe can be tuned using a reflection bridge called "magic-T". It is a device with 4 ports whose equivalent circuit is

At B and C, rf is reflected with identical efficiency. The setup involved in the tuning:

What will V be when Z_{probe} = 50 Ω?
31) Calculate the $C_1, C_2$ values that take the impedance of the following arrangement to 50 Ω, as a function of $L, r$:

![Circuit Diagram]

32) Explain the operation of the quarter-wave-based duplexer shown in page 66.

33) **Composite pulses:** Calculate the direction of the magnetization in the $x, y, z, \text{Sphere after}$

   i) A $(\pi)_x$ pulse

   ii) A $(\frac{19\pi}{20})_x$ pulse (i.e., an imperfect $\pi$-pulse)

   iii) A composite $(\pi)_x$ pulse that takes the imperfection into account:

   \[
   \left( \frac{19\pi}{40} \right)_x \left( \frac{19\pi}{20} \right)_y \left( \frac{19\pi}{20} \right)_z
   \]

Use the rotations that were worked out in the problems of Section I.
(a) Explain the operation of the 8085 microprocessor in the given context.

(b) Given that A = X + Y, compute the value of A if X = 5 and Y = 3.

(c) A computer_firstname is defined as the immediate input to a program. Use the given values to determine the computer_firstname.

\[
\begin{align*}
\text{computer_firstname} &= \text{X} + \text{Y} \\
&= 5 + 3 \\
&= 8
\end{align*}
\]