I-1) calc pop diff for:

i) ^{13}C @ 7T, 300K

$$\omega_0 = \gamma B_0 = \frac{470.8 \text{ rad}}{\text{sec}} \times 10^6$$

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{75 \times 10^6 \text{ cycles}}{\text{sec}}$$

$S = 1/2$

$k = 1.38 \times 10^{-23}$ J/K

$\gamma_C = \cancel{67.2?12}\ 67.262 \left(\frac{\text{rad}}{\text{s} \cdot \text{T}}\right) \times 10^6$

$h = 6.626 \times 10^{-34}$ J·s

$T = 300K$

$kT = 4.14e^{-21}$

Calculate excess in relation to N_T

$\cancel{2kT = 1.987 \times 10^{-3?}\text{ J}}$

$$\therefore \frac{N_\alpha - N_\beta}{N_T} =$$

$$\frac{\hbar\omega_0}{2kT} = \frac{h\nu}{2kT} =$$

$$N_\alpha - N_\beta = \frac{N_T}{2}\left(1 + \frac{\hbar\omega_0}{2kT}\right) - \frac{N_T}{2}\left(1 - \frac{\hbar\omega_0}{2kT}\right)$$

$$\gamma = \frac{q}{2m} \times g_I$$

$$\Rightarrow \frac{N_\alpha - N_\beta}{N_T} = \frac{1}{2} + \frac{\hbar\omega_0}{4kT} - \frac{1}{2} + \frac{\hbar\omega_0}{4kT}$$

$$\boxed{\frac{N_\alpha - N_\beta}{N_T} = \frac{\hbar\omega_0}{2kT} = \frac{h\nu}{2kT}} = \frac{6.626 \times 10^{-34}\ 75 \times 10^6}{2 \times 4.14e^{-21}\ \cancel{2 \times 1.987 \times 10^{-3?}}} = \frac{4.97e^{-26}}{(\quad)}$$

$=$

$$\frac{h\nu}{2kT} = 1.12e^{-5} \sim \frac{1}{10000}$$

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ii) $\cancel{\frac{25e^6 \times 6.626e^{-34}}{2 \times 4.14e^{-21}} = 2e^{-6} \approx \frac{2}{1{,}000{,}000}}$

25Mg @ 7T 300K

$\gamma =$

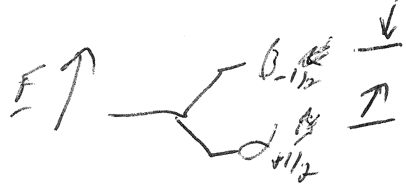
$\cancel{h\nu = 25 \text{ MHz}}$

$S = \frac{5}{2}$

ii)

$$N_j = \frac{N_T}{2S+1}\left(1 + m_j \frac{\hbar\omega_0}{kT}\right) \qquad S = \frac{5}{2} \qquad m_j = \frac{5}{2}\ \frac{3}{2}\ \frac{1}{2}\ \frac{-1}{2}\ \frac{-3}{2}\ \frac{-5}{2}$$

$$N_j = \frac{N_T}{6}\left(1 + m_j \frac{\hbar\omega_0}{kt}\right)$$



$$E_{5/2} - E_{-5/2} \Rightarrow N_{\frac{5}{2}} - N_{-\frac{5}{2}} = \frac{N_T}{6}\left(1 + \frac{5\hbar\omega_0}{2kT}\right) - \frac{N_T}{6}\left(1 - \frac{5\hbar\omega_0}{2kT}\right)$$

$$\Rightarrow \frac{\Delta N_{5/2}}{N_T} = \frac{1}{6}\left(1 - 1 + \frac{5\hbar\omega_0}{2} + \frac{5\hbar\omega_0}{2kT}\right)$$

$$= \frac{1}{6}\left(\frac{10\hbar\omega_0}{2kT}\right) = \frac{5\hbar\omega_0}{6kT}$$

or for just $\pm m = 1$

$$E_{5/2} - E_{3/2} \Rightarrow N_{\frac{5}{2}} - N_{\frac{3}{2}} = \frac{N_T}{6}\left(1 + \frac{5\hbar\omega_0}{2kT}\right) - \frac{N_T}{6}\left(1 + \frac{3\hbar\omega_0}{2kT}\right)$$

$$\frac{\Delta N}{N_T} = \frac{1}{6}\left[1 + \frac{5x}{2} - 1 - \frac{3x}{2}\right] = \frac{1}{6}x = \frac{1}{6}\left(\frac{h\omega_0}{2kT}\right)$$

$$\therefore \frac{5}{2} - \frac{3}{2} \Rightarrow \frac{1}{6}\left(\frac{6.626e^{-34} * 25e^{6}}{2 * 4.14e^{-21}}\right) = 3.3e^{-7}$$

* Note all ± 1 transitions have same pop diff

$$N_\alpha = \frac{N_T}{2}\left(1+\frac{\hbar\omega_0}{2kT}\right) \qquad N_\beta = \frac{N_T}{2}\left(1-\frac{\hbar\omega_0}{2kT}\right)$$

$E \uparrow$

$E_\beta = \frac{1}{2}\omega_0\hbar$

$E_\alpha = -\frac{1}{2}\omega_0\hbar$

$\Delta E = \omega_0$

$\vec{\mu} = \gamma\hbar\vec{S}$

$E = -\vec{\mu}\cdot\vec{B}_0 = -\gamma\hbar S_z B_0 \quad \therefore E_\alpha =$

$E = -\hbar S_z \gamma B_0$

$$\boxed{E = -\hbar S_z \omega_0}$$

$S_z|\alpha\rangle = \frac{1}{2}|\alpha \cancel{\hbar}$

$S_z|\beta\rangle = -\frac{1}{2}|\beta\rangle$

E_β $\downarrow\downarrow\downarrow\downarrow$ $N_\beta = e^{-E_\beta/kT}$

E_α $\uparrow\uparrow\uparrow\uparrow\uparrow$ $N_\alpha = e^{-E_\alpha/kT}$

$$\frac{N_\beta}{N_\alpha} = \frac{e^{-E_\beta/kT}}{e^{-E_\alpha/kT}} = e^{-(E_\beta-E_\alpha)/kT} = e^{\hbar\omega_0/kT}$$

$$\frac{N_\beta}{N_\alpha} = e^{-\hbar\omega_0/kT}$$

expand

$$\frac{N_\beta}{N_\alpha} = e^{-\hbar\omega_0/kT} = 1 - \frac{\hbar\omega_0}{kT}$$

$N_T = N_\alpha + N_\beta$

$N_\alpha = N_T - N_\beta$

$$N_\beta = N_\alpha\left(1-\frac{\hbar\omega_0}{kT}\right)$$

$$N_\beta = (N_T - N_\beta)\left(1-\frac{\hbar\omega_0}{kT}\right)$$

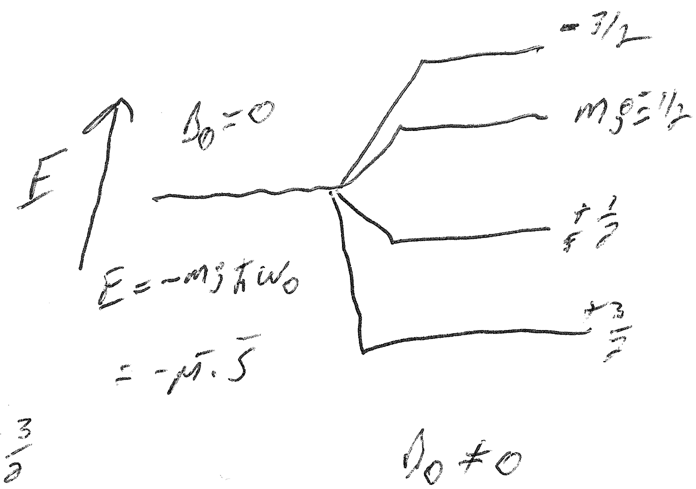
$$= N_T - N_T\left(\frac{\hbar\omega_0}{k}\right) - N_\beta + N_\beta\frac{\hbar\omega_0}{kT}$$

$$2N_\beta = N_T - N_T\left(\frac{\hbar\omega_0}{kT}\right) = \cancel{N_T}$$

$$\therefore \boxed{\begin{aligned} N_\beta &= \frac{N_T}{2}\left(1-\frac{\hbar\omega_0}{2kT}\right) \\ N_\alpha &= \frac{N_T}{2}\left(1+\frac{\hbar\omega_0}{2kT}\right)\end{aligned}}$$

General Form:

$M_s \quad (-S \leq m_s \leq S)$



$E = -m_s \hbar \omega_0$

$= -\vec{\mu} \cdot \vec{S}$

$S = \frac{3}{2}$

$m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

General Pop eqn

$$\boxed{N_j = \frac{N_T}{2S+1}\left(1 + \frac{m_s \hbar \omega_0}{kT}\right)}$$

ω

I-7) $\frac{\Delta N}{N_T}(C) = \frac{1}{10000}$ @298K

$T = 298K$

$\nu_{Mg} = 25MHz$

$\nu_C = 75\ MHz$

$\cancel{KT = 4.1e^{-21}}$

$h = 6.626e^{-34}\ Js$

$k = 1.38e^{-23}\ J/K$

at what Temp Does The Mg Sample Need to be to have same $\frac{\Delta N}{N}$ @($\pm 1/2$) Transition? Note, Mg = 10*C in Natural abudance

@ 300k Mg $\frac{3/2}{5/2}$ => $\frac{\Delta N}{N_T} = 3.3e^{-7}$

$$\Delta N_{1/2} = \frac{N_T}{6}\left(1 + \frac{\hbar w}{2kT}\right) - \frac{N_T}{6}\left(1 - \frac{\hbar w}{2kT}\right)$$

$$\frac{\Delta N_{1/2}}{N_T} = \frac{\hbar w_0}{6KT} \Big\} Mg\ (\tfrac{1}{2} - {}^{-1}\!/_2)$$

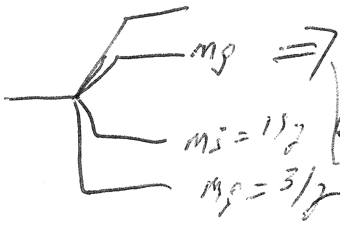
$$\frac{\Delta N_{1/2}}{N_C} = \frac{\hbar w_0}{2KT} \Big\} C$$

$$\frac{10 * h\nu}{6KT} = \frac{1}{10,000} \qquad T = \frac{10^5 h\nu}{6K} = \frac{1.66e^{-21}}{6K} = \cancel{2K}\ 20K$$

check $\frac{\Delta N_{1/2}}{N_T} = \frac{h \cdot 25e^6}{6KT} = \frac{6.626e^{-34} * 25e^6}{6 * \cancel{4.1e^{-21}}\ K*20} = \frac{1.656e^{-26}}{\cancel{12K}\ 120k} = \frac{1}{\cancel{10,000}0\ 100,000}$

*10 Nat Abund diff => $\frac{\Delta n_{1/2}}{N_T} = \frac{1}{10,000}$ ✓ @ 20K

III-3) start from here:



$$\Rightarrow \boxed{N_j = \frac{N_T}{2S+1}\left(1 + \frac{m_j \hbar \omega_0}{KT}\right)}$$

eg) $S = 3/2$

$m_j = (-S \le m_j \le S)$

$m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

$\omega = \gamma B_0$

$E = -\vec{\mu} \cdot \vec{B}$

$\mu = \gamma \hbar S_z$

$\therefore E = -\gamma \hbar B_0 S_z$

$$\boxed{E_j = -\hbar \omega_0 S_z}$$

$$\boxed{\vec{M}_0 = \sum_j \bar{\mu}_j \cdot N_j}$$

$$\boxed{\begin{aligned}\mu &= \gamma \hbar S_z \\ &= \cancel{\gamma m_j \hbar S_z}\end{aligned}}$$

$\bar{M}_0 =$

$$\boxed{\begin{aligned}\mu &= \gamma \hbar S_z \\ &= m_j \hbar \gamma\end{aligned}}$$

$$\vec{M}_0 = \sum_j (m_j \gamma \hbar)\left[\frac{N_T}{2S+1}\left(1 + \frac{m_j \hbar \omega_0}{KT}\right)\right]$$

$$= \sum_j \left[\cancel{\frac{N_T(m_j \gamma \hbar)}{2S+1}}^{0} + N_T\frac{(m_j \gamma \hbar)(m \hbar \gamma B_0)}{(2S+1) KT}\right]$$

Note $\sum_j N_T m_j \gamma \hbar = c\, m_j = -\frac{1}{2} + \frac{1}{2} = 0 \;\therefore$ cancels out

$= -\frac{3}{2} + -\frac{1}{2} + \frac{1}{2} + \frac{3}{2} = 0$

$$\therefore \boxed{M_0 = \sum_j \left(\frac{N_T}{2S+1}\right)\left(\frac{m_j^2 \gamma^2 \hbar^2 B_0}{KT}\right)}$$

$s = 1/2 \Rightarrow M_0 \sum_{s} \frac{N_T}{2}\left[m_s^2 (c)\right]$

$M_0 = \cancel{}\frac{N_T}{2}\left[\left(\frac{1}{2}\right)^2 c + \left(\frac{-1}{2}\right)^2\right] = \frac{N_T}{2}\left[\frac{1}{4}c + \frac{1}{4}c\right]$

$= \frac{N_T}{2}\left(\frac{c}{2}\right) = \frac{N_T}{2}\left[\frac{\gamma^2 \hbar^2 b_0}{2kT}\right]$

$c = \frac{\gamma^2 \hbar^2 b_0}{kT}$

$$\boxed{M_0(1/2) = \frac{N_T \gamma^2 \hbar^2 b_0}{4kT}}$$

$\llcorner$ Curie's Law

$M_0 \Rightarrow$ linear depend on
N_T, b_0

$\Rightarrow$ inversely to temp

$\Rightarrow$ Quadratic to γ!

$c = \frac{N_T \gamma^2 \hbar b_0}{(2s+1)kT}$

$s = \frac{3}{2}$ $\quad M_0 = \sum_{s} c\, m_s^2$

$= \left[\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 + \left(\frac{-3}{2}\right)^2\right] c$

$= \left[\frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4}\right] c$

$= \frac{20}{4} c = 5c \cancel{[} = 5\left[\frac{N_T \gamma^2 \hbar^2 b_0}{(2s+1)kT}\right]$

$$\boxed{\vec{M}_{0\left(\frac{3}{2}\right)} = \frac{5}{4}\left[\frac{N_T \gamma^2 \hbar^2 b_0}{kT}\right]}$$

$2\left(\frac{3}{2}\right) + 1 = 4$

$\llcorner$ 5 x's ~~more sensitive~~ than $s = \frac{1}{2}$
Greater lot diff

I-4) $\langle m|\hat{A}|n\rangle = (\langle n|\hat{A}|m\rangle)^*$ $\qquad \hat{O} = \hat{O}^{T*}$

$O_{ij} = (O_{ji})^*$ (Transpose & cc)

$O_{ij} = (O_{ji})^* \Rightarrow$ is Real/Hermitian $\qquad \langle i|\hat{O}|j\rangle = O_{ij}$

1) $\langle i|\hat{O}|j\rangle = a_j \langle i|j\rangle$

NOTE $\langle i| = |i\rangle^*$

2) $(\langle j|\hat{O}|i\rangle)^* = \langle i|\hat{O}^*|j\rangle$

$\langle j| = |j\rangle^*$

$= a_j^* \langle i|j\rangle$

$\langle j|^* = |j\rangle$

$\delta_{ij} = \langle i|j\rangle$

$\delta_{ij} = 1 \quad (i=j)$

$= 0 \quad (i \neq j)$

$a_j\langle i|j\rangle = a_j^* \langle i|j\rangle$

$(a_j - a_j^*)\,\delta_{ij} = 0$

i must $= j$ else $\rightarrow \emptyset$

$\therefore a_j = a_j^*$

real

I-4) ii) which of these ops is observable?

$I^2,\ I_z,\ I_x,\ I_y$

* use matrix (Pauli) & check transpose/cc

$$I^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix} \qquad I^{2\,*+} = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix}$$ ✓

$$I_{\cancel{z}} = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} \qquad I_z^{+*} = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$ ✓

$$I^+ = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix} \qquad I^{+(+*)} = \hbar\begin{pmatrix}0&0\\1&0\end{pmatrix}$$ NO

$$I_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad I^{+*} = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$ ✓

I-5) calculate

$[I^2, I^+]$

$I^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $I^+ = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$I_z = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $I_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$

Let $\hbar = 1$

$$[I^2, I^+] = I^2 I^+ - I^+ I^2$$
$$= \frac{3}{4}\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]$$
$$= \frac{3}{4}\left[\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&1\\0&0\end{pmatrix}\right] = 0$$

$$[I^2, I_x] = \frac{3}{4}\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} - \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]$$
$$= \frac{3}{4}\left[\begin{pmatrix}0&1\\1&0\end{pmatrix} - \begin{pmatrix}0&1\\1&0\end{pmatrix}\right] = 0$$

$$[I^2, I^+] = \frac{3}{4}\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}\right]$$
$$= \frac{3}{4}\left[\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&1\\0&0\end{pmatrix}\right] = 0$$

$$[I_z, I^+] = \frac{1}{2}\left[\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right]$$
$$= \frac{1}{2}\left[\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&-1\\0&0\end{pmatrix}\right] = \begin{pmatrix}0&1\\0&0\end{pmatrix} = I^+$$

I-6) Show that $\boxed{I^2 = I_z^2 + \frac{1}{2}(I_+I_- + I_-I_+)}$

use $I_+ = I_x + iI_y$ $\quad I_+ + I_- = 2I_x \quad I_+ - I_- = 2iI_y$

$I_- = I_x - iI_y \quad I_x = \frac{I_+ + I_-}{2} \quad I_y = \frac{I_+ - I_-}{2i}$

$I_+ I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad I^2 = I_z^2 + I_x^2 + I_y^2$

$$I_x^2 = \frac{1}{4}\left[(I_+ + I_-)(I_+ + I_-)\right] = \frac{1}{4}\left[\cancel{I_+I_+} + I_+I_- + I_-I_+ + \cancel{I_-I_-}\right]$$

$$I_y^2 = -\frac{1}{4}\left[(I_+ - I_-)(I_+ - I_-)\right] = \frac{-1}{4}\left[I_+I_+ - I_+I_- - I_-I_+ + I_-I_-\right]$$

$$= \frac{1}{4}\left[-\cancel{I_+I_+} + I_+I_- + I_-I_+ - \cancel{I_-I_-}\right]$$

$$I_x^2 + I_y^2 = \frac{1}{4}\left[2I_+I_- + 2I_-I_+\right] = \frac{I_+I_- + I_-I_+}{2}$$

$$\therefore \boxed{I^2 = I_z^2 + \frac{I_+I_- + I_-I_+}{2}}$$

I-7) Demonstrate that if 2 Hermitian ops A, B fulfill

$[\hat{A},\hat{B}]=0$, then they possess a common basis of E's (ie share same ψ_i^o)

$\hat{A}\hat{B}-\hat{B}\hat{A}=0$

somewhat of a logic question.

* see Quantum Notes Prob set 4-4!

$\hat{A}\psi_i^o = a_i^o \psi_i^o$

Since $[\hat{A},\hat{B}]=0$

$[\hat{A},\hat{B}]\psi_i^o = 0$

$\hat{A}\hat{B}\psi_i^o = \hat{B}(\hat{A}\psi_i^o)$

$\hat{A}\hat{B}\psi_i^o = a_i^o(\hat{B}\psi_i^o)$

$\hat{A}(\hat{B}\psi_i^o) = a_i^o(\hat{B}\psi_i^o)$ using

$\hat{A}\psi_i^o = a_i^o\psi_i^o$

we have $\hat{A}\hat{B}\psi_i^o = \underset{\text{EV}}{a_i^o}\underbrace{\hat{B}\psi_i^o}_{\text{The EF now}}$. we started with $\hat{A}\psi_i^o = a_i^o\psi_i^o$.

$\therefore$ $B\psi_i$ must be at most a constant * ψ_i (Follow from $\hat{A}\psi_i = a_i\psi_i$)

ie) $B\psi_i^o = c\psi_i^o$ } This is EV eqn for $\hat{B}$ w/ EF ψ_i^o though

$\therefore$ $[A,B]$ share same ψ_i^o

Hermitian ops 1) have real EV's

2) share sam EF ψ

I-8) $\boxed{\hat{A}\psi_i = a_i \psi_i}$

See standards prob set 1-5

calc expectation value $\int \psi_j^* \hat{A} \psi_i = a_i \int \psi_j^* \psi_i$

If $\hat{A}$ is Hermitian ← substitute

can show this → $\int \psi_j^* \hat{A} \psi_i = \int (\hat{A}\psi_j)^* \psi_i$

using $\hat{A}\psi_i = a_i \psi_i$ & $\hat{B}\psi_j = a_j \psi_j$ are key here

substitute

use Hermitian identity too

$$\int (\hat{A}\psi_j)^* \psi_i = a_i \int \psi_j^* \psi_i$$

$\boxed{\text{Now} \quad \hat{A}\psi_j = a_j \psi_j}$

$$\int (\hat{A}\psi_j)^* \psi_i = \int (a_j \psi_j)^* \psi_i = \int a_j^* \psi_j^* \psi_i$$

real #

$a_j^* = a_i$

$$\therefore a_j^* \int \psi_j^* \psi_i = a_i \int \psi_j^* \psi_i$$

∴ is real

$a_j^* = a_i$ { a_i = its complex conjug ∴ real i.e) no i term

$$= (a_j^* - a_i) \int \psi_j^* \psi_i = 0$$

$$= (a_j - a_i) \int \psi_j^* \psi_i = 0$$

if $a_j \neq a_i$ Then $\int \psi_j^* \psi_i$ must $= 0$ ∴ are orthogonal

$$\int \psi_j^* \psi_i = \delta_{ij} \begin{cases} \delta_{ij} = 1 \ (i = j) \\ \delta_{ij} = 0 \ (i \neq j) \end{cases}$$

	I_x	I_y	I_z	I^2	I^+	I^-
I_x	$\frac{I^2}{3}$	$\frac{i}{2}I_z$	$\frac{-i}{2}I_y$	$+\frac{3}{4}I_x$	$\frac{I^2}{3}-\frac{I_z}{2}$	$\frac{I^2}{3}+\frac{I_z}{2}$
I_y	$\frac{-i}{2}I_z$	$\frac{I^2}{3}$	$\frac{i}{2}I_x$	$\frac{3}{4}I_y$	$i\left(\frac{I^2}{3}-\frac{I_z}{2}\right)$	$i\left(\frac{I^2}{3}+\frac{I_z}{2}\right)$
I_z	$\frac{i}{2}I_y$	$\frac{-i}{2}I_x$	$\frac{I^2}{3}$	$\frac{3}{4}I_z$	$\frac{1}{2}I^+$	$\frac{-1}{2}I^-$
I^2	$\frac{3}{4}I_x$	$\frac{3}{4}I_y$	$\frac{3}{4}I_z$	$\frac{3}{4}I^2$	$\frac{3}{4}I^+$	$\frac{3}{4}I^-$
I^+	$\frac{I^2}{3}+\frac{I_z}{2}$	$i\left(\frac{I^2}{3}+\frac{I_z}{2}\right)$	$\frac{-1}{2}I^+$	$\frac{3}{4}I^+$	0	$\frac{2}{3}I^2+I_z$
I^-	$\frac{I^2}{3}-\frac{I_z}{2}$	$i\left(\frac{I^2}{3}-\frac{I_z}{2}\right)$	$\frac{1}{2}I^-$	$\frac{3}{4}I^{\cancel{+}-}$	$\frac{2}{3}I^2-I_z$	0

above = R·C

let $\hbar = 1$

$I_z = \frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\qquad I^+ = \begin{pmatrix}0&1\\0&0\end{pmatrix}$ $\qquad I^+ = I_x + iI_y$

$I^- = \begin{pmatrix}0&0\\1&0\end{pmatrix}$ $\qquad I^- = I_x - iI_y$

$I_x = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$ $\qquad I^2 = \begin{pmatrix}1&0\\0&1\end{pmatrix}\frac{3}{4}$ $\qquad |\alpha\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$I_y = \frac{+i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ $\qquad |\beta\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$[S_i, S_j] = i\epsilon_{ijk}S_k$ $\qquad \epsilon_{ijk} = \begin{cases}1 & ijk = xyz\\ -1 & \text{otherwis}\end{cases}$

eg) $I_x I_y = iI_z$

I-9)

$$I_x I_x = \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = \frac{1}{4}\mathbb{1} = \frac{I^2}{3}$$

$$\frac{3}{4} I^2 = \mathbb{1}\frac{3}{4}$$

$$\frac{4}{3} I^2 = \mathbb{1}$$

$$I_x I_y = \frac{i}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix} = \frac{i}{4}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{i}{2} I_z$$

$$-i^2 = 1$$

$$-i \cdot i = 1$$

$$I_x I_z = \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\frac{1}{2} = \frac{1}{8}\begin{pmatrix}0&-1\\1&0\end{pmatrix} = \frac{1}{4}\,\frac{1}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}$$

$$= \frac{1}{4}\, -i \cdot \frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}$$

$$= \frac{-i}{4} I_y \checkmark$$

$$I_x I_z^2 = \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{3}{4}\begin{pmatrix}1&0\\0&1\end{pmatrix} = \frac{3}{16}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \frac{3}{4} I_x \checkmark$$

$$I_x I^+ = \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0&0\\0&1\end{pmatrix} = \frac{1}{4} I$$

$$= \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\left[\frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\frac{i}{2}\right.$$

$$= \frac{1}{16}(\mathbb{1}) + \frac{i}{8}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{1}{16}\mathbb{1} + \frac{i}{4}\,\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\frac{i I_z}{4}$$

$$I_x I^+ = \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{1}{4} + i\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\frac{i}{2}$$

$$= \frac{1}{16}\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{-1}{8}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\frac{1}{16}\mathbb{1} - \frac{1}{4}\,\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{1}{16}\mathbb{1} - \frac{1}{4} I_z \checkmark$$

$$I_x I_- = I_x I_x - i I_x I_y$$

$$= \frac{\mathbb{1}}{16} - \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\frac{i \cdot i}{2} = -\frac{i}{4}\begin{pmatrix}1&0\\0&-1\end{pmatrix} i = \frac{\mathbb{1}}{16} - \frac{i}{2}\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} i = \frac{\mathbb{1}}{16} + i\frac{I_z}{2}$$

$$I_x I_+ = I_x I_x + i I_x I_y$$

$$= \frac{\mathbb{1}}{4} + i\,\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix} = \frac{\mathbb{1}}{4} + (-1)\left(\frac{1}{4}\right)\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$= \frac{\mathbb{1}}{4} - \frac{1}{2}\,\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\frac{3\mathbb{1}}{4} = I^2 \qquad = \frac{\mathbb{1}}{4} - \frac{I_z}{2} = \frac{I^2}{3} - \frac{I_z}{2}$$

$$\mathbb{1} = \frac{4I^2}{3}$$

$I_x I_-$ = same as above sign change 2nd term

$$I_x I_- = \frac{I^2}{3} + \frac{I_z}{2}$$

$$I_x I^2 = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}\frac{3}{4} = \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$I_y I_x = \frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{1}{2} = \frac{i}{4}\begin{pmatrix}-1&0\\0&1\end{pmatrix} = \frac{-i}{2}\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$= -\frac{I_z}{2} i$$

$$I_y I_y = \frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix} = \frac{-1}{4}\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$$

$$= \frac{1}{4}\mathbb{1} = \frac{I^2}{3} \qquad \cancel{\tfrac{3}{2}} I^2 = \frac{3}{4}\mathbb{1} \ldots$$

$4/2 = 3$

$$I^- I^+ = \cancel{\begin{pmatrix}0&1\\0&\end{pmatrix}} = \begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} - \begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$(I_x - iI_y)(I_x + iI_y) = I_x I_x + i I_x I_y \cancel{+} - i I_y I_x + I_y I_y$$

$$\frac{I^2}{3} + i\left(\frac{iI_z}{2}\right) - i\left(\frac{-i}{2}I_z\right) + \frac{I^2}{3}$$

$$\frac{I^2}{3} - \frac{I_z}{2} - \frac{I_z}{2} + \frac{I^2}{3}$$

$$\frac{2}{3}I^2 - I_z$$

$$I^+ I^- = (I_x + iI_y)(I_x - iI_y)$$

$$= I_x I_x - i I_x I_y + i I_y I_x - i^2 I_y I_y$$

$$= \frac{I^2}{3} - i\frac{i}{2}I_z \quad + i\left(\frac{-i}{2}I_z\right) + \frac{I^2}{3}$$

$$= \frac{2}{3}I^2 + \frac{I_z}{2} + \frac{I_z}{2} = \frac{2}{3}I^2 + I_z$$

I-10) $I_z = \frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\quad I_z|\alpha\rangle = \frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}|\alpha\rangle$

$I_z|\beta\rangle = \frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\-1\end{pmatrix} = \frac{-1}{2}|\beta\rangle$

$$\langle I_z\rangle = \det\begin{vmatrix}\langle\alpha|I_z|\alpha\rangle-\lambda & \langle\alpha|I_z|\beta\rangle\\ \langle\beta|I_z|\alpha\rangle & \langle\beta|I_z|\beta\rangle-\lambda\end{vmatrix} = 0 \qquad \begin{vmatrix}\frac{1}{2}-\lambda & 0\\ 0 & \frac{-1}{2}-\lambda\end{vmatrix} = 0$$

$$\left(\frac{1}{2}-\lambda\right)\left(\frac{-1}{2}-\lambda\right) = 0 \qquad \frac{-1}{4}+\frac{\lambda}{2}-\frac{\lambda}{2}+\lambda^2 = 0$$

$$\lambda^2 - \frac{1}{4} = 0 \quad \therefore \quad \lambda = \pm\frac{1}{2}$$

$\lambda = \frac{1}{2}$ $\quad \begin{pmatrix}\frac{1}{2}-\frac{1}{2} & 0\\ 0 & \frac{-1}{2}-\frac{1}{2}\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}0&0\\0&-1\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}$ $\Big\{ -1c_2 = 0$

$c_2 = 0 \;\therefore\; c_1 = 1$

Not sure about this ☺ $\Rightarrow$ $c_1^2 + 0^2 = 1$ $\qquad \psi = |\alpha\rangle$

$\lambda = \frac{-1}{2}$ $\quad \begin{pmatrix}\frac{1}{2}-(\frac{-1}{2}) & 0\\ 0 & \frac{-1}{2}-(\frac{-1}{2})\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}$ $\Big\{ c_1 = 0$

$\therefore c_2 = 1$

$\cancel{c_1^2}^{\,0} + c_2^2 = 1 \qquad \psi = |\beta\rangle$

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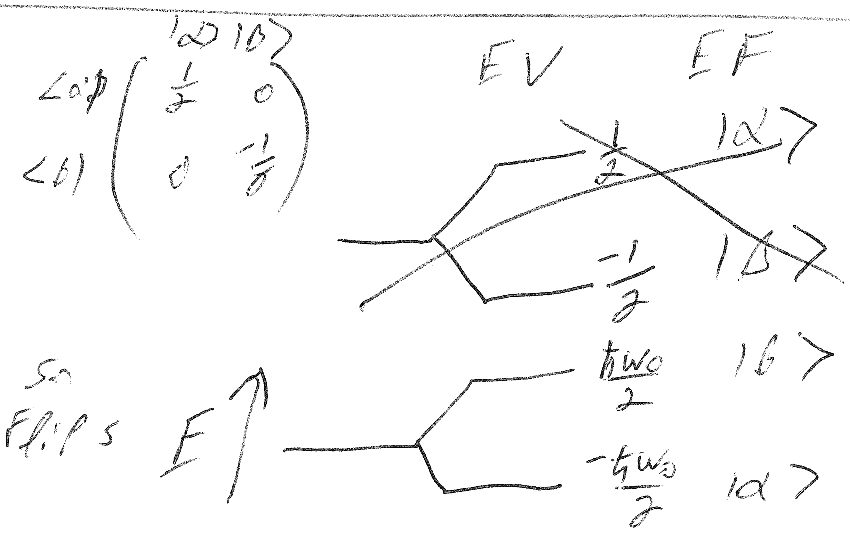
* could just note its a diag matrix & read off EV & EF

$$\begin{array}{c|cc} & |\alpha\rangle & |\beta\rangle \\ \hline \langle\alpha| & \frac{1}{2} & 0 \\ \langle\beta| & 0 & \frac{-1}{2}\end{array}$$



$E = -\mu \cdot B$

$\mu = \gamma\hbar I_z B$

$=$

$\Delta E = \hbar\omega_0 = h\nu$

I-10) $I_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$I_x|\alpha\rangle = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2}|\beta\rangle$

$I_x|\beta\rangle = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}|\alpha\rangle$

$$\langle I_x\rangle = \det\begin{vmatrix} \langle\alpha|I_x|\alpha\rangle - \lambda & \langle\alpha|I_x|\beta\rangle \\ \langle\beta|I_x|\alpha\rangle & \langle\beta|I_x|\beta\rangle - \lambda \end{vmatrix} = \begin{vmatrix} -\lambda & \frac{1}{2} \\ \frac{1}{2} & -\lambda \end{vmatrix} = 0$$

$\lambda^2 - \frac{1}{4} = 0 \quad \therefore \lambda = \pm\frac{1}{2}$ } EVs

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$\lambda = \frac{1}{2}$ $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ $\begin{cases} -\frac{c_1}{2} + \frac{c_2}{2} = 0 \\ \frac{c_1}{2} - \frac{c_2}{2} = 0 \end{cases}$ $\frac{c_1}{2} = \frac{c_2}{2}$

$c_1 = c_2$

$\therefore c_1^2 + c_2^2 = 1 \quad 2c_1^2 = 1 \quad c_1 = \frac{1}{\sqrt{2}} \quad \therefore c_2 = \frac{1}{\sqrt{2}}$

$\psi = \frac{1}{\sqrt{2}}(|\alpha\rangle + |\beta\rangle)$

$e_1' = (c, c)$ for all EFs actually w EV = 1/2

$\lambda = -\frac{1}{2}$ $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ $\begin{cases} \frac{c_1}{2} + \frac{c_2}{2} = 0 \\ \frac{c_1}{2} + \frac{c_2}{2} = 0 \end{cases}$ $c_1 = -c_2$

$c_1^2 + (-c_2)^2 = 1 \quad c_1 = \frac{1}{\sqrt{2}} \quad c_2 = -\frac{1}{\sqrt{2}}$

$\therefore \psi = \frac{1}{\sqrt{2}}(|\alpha\rangle - |\beta\rangle)$ with EV $= -\frac{1}{2}$

$= \frac{1}{\sqrt{2}}(|\beta\rangle - |\alpha\rangle)$

$E = -\vec{\mu}\cdot\vec{B}$ $\therefore$ Flips actually Not

$\langle I_x\rangle$	EV	EF
	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}(|\alpha\rangle + |\beta\rangle)$
	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}(|\alpha\rangle - |\beta\rangle)$

I-10) $I_y = \frac{i}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad I_y|\alpha\rangle = \frac{i}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{i}{2}|\beta\rangle$

$I_y|\beta\rangle = \frac{i}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{i}{2}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \frac{-i}{2}|\alpha\rangle$

$\langle I_y \rangle = \det \begin{vmatrix} \langle\alpha|I_y|\alpha\rangle - \lambda & \langle\alpha|I_y|\beta\rangle \\ \langle\beta|I_y|\alpha\rangle & \langle\beta|I_y|\beta\rangle - \lambda \end{vmatrix} = 0 \qquad \begin{vmatrix} 0-\lambda & -i/2 \\ \frac{i}{2} & 0-\lambda \end{vmatrix} = 0$

$\det \begin{vmatrix} -\lambda & -i/2 \\ \frac{i}{2} & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 = \left(\frac{-i}{2}\right)\left(\frac{i}{2}\right) \Rightarrow \lambda^2 - \left(\frac{-i^2}{4}\right)$

$= \lambda^2 - \frac{1}{4} \quad \therefore \boxed{\lambda = \pm\frac{1}{2}}$

$\lambda = \frac{1}{2} \quad \begin{pmatrix} -\frac{1}{2} & \frac{i}{2} \\ \frac{i}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \begin{cases} \frac{-c_1}{2} + \frac{ic_2}{2} = 0 \\ \frac{ic_1}{2} - \frac{c_2}{2} = 0 \end{cases} \quad \therefore c_1 = ic_2, \quad ic_1 = c_2$

$\boxed{e_1' = (c, ic) \text{ are EFS w/ EV} = \frac{1}{2}}$

$i = \sqrt{-1}$ $\qquad$ Just use $c_1 = c_2$ thru $c_1^2 + c_2^2 = 1$ $\quad \therefore c_1 = \frac{1}{\sqrt{2}}$ & $c_2 = \frac{1}{\sqrt{2}}i$

$i^2 = -1$

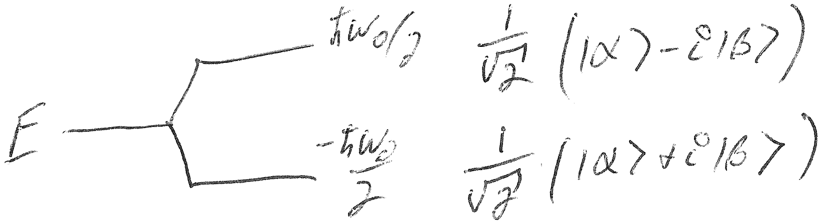
$\Psi = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + i|\beta\rangle\right)$

$\lambda = \frac{-1}{2} \quad \begin{pmatrix} \frac{1}{2} & \frac{i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \begin{cases} \frac{c_1}{2} + \frac{ic_2}{2} = 0 \\ \end{cases} \quad \therefore c_1 = -ic_2$

Just use $c_1 = -c_2$ for Norm

$\therefore c_1^2 + c_2^2 = 1 \Rightarrow c_1 = \frac{1}{\sqrt{2}}$ & $c_2 = \frac{-i}{\sqrt{2}}$

$\boxed{\Psi = \frac{1}{\sqrt{2}}\left(|\alpha\rangle - i|\beta\rangle\right)}$



$E = -\mu_z \cdot B_0$

$\lambda = \frac{1}{2}$ $\begin{pmatrix} -\frac{1}{2} & \frac{i}{2} \\ \frac{i}{2} & -\frac{1}{2} \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix}$ $\begin{cases} -\frac{c_1}{2} + \frac{ic_2}{2} = 0 \\ \therefore c_1 = ic_2 \end{cases}$

$c_1^2 + c_2^2 = 1$ $\quad$ $\therefore$ vectors $e_i = (c, ic)$ form are efs w/ev $= \frac{1}{2}$

$(ic_2)^2 + c_2^2 = 1$

eg) $(1, i)$ $(|\alpha\rangle, i|\beta\rangle)$

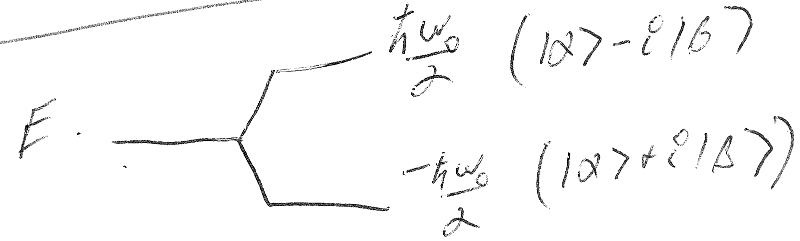
$\Psi = (|\alpha\rangle + i|\beta\rangle)\frac{1}{\sqrt{2}}$

$\lambda = -\frac{1}{2}$ $\begin{pmatrix} \frac{1}{2} & \frac{i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ $\begin{cases} \frac{c_1}{2} + \frac{ic_2}{2} = 0 \end{cases}$ Just use $c_1 = -c_2$

$c_1 = -ic_2$

vectors of form $e_i = (c, -ic)$ are efs w/ef $= -\frac{1}{2}$

$\Psi = (|\alpha\rangle - i|\beta\rangle)\frac{1}{\sqrt{2}}$



$E = -\mu_z B_0$

I-10 $\alpha\alpha \quad \alpha\beta$
$\beta\alpha \quad \beta\beta$

$$I_x|\alpha\rangle = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{2}|\beta\rangle$$

$$I_x|\beta\rangle = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}|\alpha\rangle$$

$$\begin{pmatrix}\langle\alpha|I_x|\alpha\rangle-\lambda & \langle\alpha|I_x|\beta\rangle\\ \langle\beta|I_x|\alpha\rangle & \langle\beta|I_x|\beta\rangle-\lambda\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0$$

$$\begin{pmatrix}\frac{1}{2}\langle\alpha|\beta\rangle-\lambda & \frac{1}{2}\langle\alpha|\alpha\rangle\\ \frac{1}{2}\langle\beta|\beta\rangle & \frac{1}{2}\langle\beta|\alpha\rangle-\lambda\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0 \qquad \begin{pmatrix}0&\frac{1}{2}\\ \frac{1}{2}&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0$$

$$c_1^2+c_2^2=1$$

$$\frac{1}{2}c_2+\frac{1}{2}c_1=0 \qquad c_1=-c_2 \qquad c_1=\frac{1}{\sqrt{2}} \quad c_2=\frac{-1}{\sqrt{2}}$$

$$EVs=\langle I_x\rangle=\frac{1}{2}=\lambda$$

$$\psi=\frac{1}{\sqrt{2}}(|\alpha\rangle-|\beta\rangle) \text{ or } \frac{1}{\sqrt{2}}(|\beta\rangle-|\alpha\rangle)$$

$$I_y|\alpha\rangle = \frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{-i}{2}\begin{pmatrix}0\\1\end{pmatrix} = \frac{-i}{2}|\beta\rangle$$

$$I_y|\beta\rangle = \frac{i}{2}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{-i}{2}\begin{pmatrix}1\\0\end{pmatrix} = \frac{-i}{2}|\alpha\rangle$$

$$\langle I_y\rangle \Rightarrow \begin{pmatrix}\langle\alpha|I_y|\alpha\rangle & \langle\alpha|I_y|\beta\rangle\\ \langle\beta|I_y|\alpha\rangle & \langle\beta|I_y|\beta\rangle\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0$$

$$\begin{pmatrix}0&-i/2\\ -\frac{i}{2}&0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=0$$

$$c_1^2+c_2^2=0 \Rightarrow c_1=\frac{1}{\sqrt{2}}$$
$$c_2=\frac{-1}{\sqrt{2}}$$

$$-\frac{ic_1}{2}-\frac{ic_2}{2}=0 \qquad c_1=-c_2$$

$$\langle I_y\rangle=\frac{-i}{2}$$

$$\psi=\frac{1}{\sqrt{2}}(|\alpha\rangle-|\beta\rangle) \quad \text{or } \frac{1}{\sqrt{2}}(|\beta\rangle-|\alpha\rangle)$$

I-11) The es^s of $S=1$ are: $I_z|+1\rangle = \hbar|+1\rangle$

$I_z|0\rangle = 0$

$I_z|-1\rangle = -\hbar|-1\rangle$

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use: $I_+|+1\rangle = 0$ $\quad I_-|+1\rangle = \hbar\sqrt{2}|0\rangle$

$I_+|0\rangle = \hbar\sqrt{2}|+1\rangle$ $\quad I_-|0\rangle = \hbar\sqrt{2}|-1\rangle$

$I_+|-1\rangle = \hbar\sqrt{2}|0\rangle$ $\quad I_-|-1\rangle = 0$

$\langle\ \rangle\langle\ \rangle$

Calculat I_x I_y I_z I^2 in matrix form $\langle I_x\rangle =$

$$\begin{array}{cc} & \begin{array}{ccc} |1\rangle & |0\rangle & |-1\rangle \end{array} \\ \begin{array}{c} \langle 1| \\ \langle 0| \\ \langle -1| \end{array} & \begin{pmatrix} \langle 1|I_x|1\rangle & \langle 1|I_x|0\rangle & \langle 1|I_x|-1\rangle \\ \langle 0|I_x|1\rangle & \langle 0|I_x|0\rangle & \langle 0|I_x|-1\rangle \\ \langle -1|I_x|1\rangle & \langle -1|I_x|0\rangle & \langle -1|I_x|-1\rangle \end{pmatrix} \end{array} = \langle I_x\rangle$$

etc.

I-1 a)

$$I_z = \begin{array}{c|ccc} & 1 & 0 & -1 \\ \hline 1 & \langle 1|I_z|1\rangle & \langle 1|I_z|0\rangle & \langle 1|I_z|-1\rangle \\ 0 & \langle 0|I_z|1\rangle & \langle 0|I_z|0\rangle & \langle 0|I_z|-1\rangle \\ -1 & \langle -1|I_z|1\rangle & \langle -1|I_z|0\rangle & \langle -1|I_z|1\rangle \end{array}$$

$$\boxed{\begin{aligned} I_z|1\rangle &= \hbar|1\rangle \\ I_z|0\rangle &= 0 \\ I_z|-1\rangle &= \hbar|-1\rangle \end{aligned}}$$

$$\langle I_z\rangle = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix} = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$I^2 = I_x^2 + I_y^2 + I_z^2$$

$$\cancel{I_x^2 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 0\cdot\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - 1\cdot\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + 0\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

See other Page

$I_+ = I_x + iI_y$ $\boxed{I_x}$; $I_x = \frac{I_+ + I_-}{2}$ $\quad I_y = \frac{I_x - I_y}{2i}$

$I_- = I_x - iI_y$

$$I_x|1\rangle = \frac{I_+|1\rangle + I_-|1\rangle}{2} = \frac{\sqrt{2}|0\rangle}{2} = \frac{|0\rangle}{\sqrt{2}}$$

$$I_x|0\rangle = \frac{I_+|0\rangle + I_-|0\rangle}{2} = \frac{\sqrt{2}|1\rangle + \sqrt{2}|-1\rangle}{2} = \frac{|1\rangle + |-1\rangle}{\sqrt{2}}$$

$$I_x|-1\rangle = \frac{1}{2}\left(I_+|-1\rangle + I_-|-1\rangle\right) = \frac{\sqrt{2}|0\rangle}{2} = \frac{|0\rangle}{2}$$

Note $\langle 1|0\rangle = 0$. etc

$\langle 1|-1\rangle = 0$

$\langle 1|1\rangle = 1$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{array}{c} 1 \\ 0 \\ -1 \end{array} & \begin{pmatrix} \frac{\langle 1|0\rangle}{\sqrt{2}} & \left(\frac{\langle 1|1\rangle}{\sqrt{2}} + \frac{\langle 1|-1\rangle}{\sqrt{2}}\right) & \frac{\langle 1|0\rangle}{\sqrt{2}} \\ \frac{\langle 0|0\rangle}{\sqrt{2}} & \left(\frac{\langle 0|1\rangle}{\sqrt{2}} + \frac{\langle 0|-1\rangle}{\sqrt{2}}\right) & \frac{\langle 0|0\rangle}{\sqrt{2}} \\ \frac{\langle -1|0\rangle}{\sqrt{2}} & \left(\frac{\langle -1|1\rangle}{\sqrt{2}} + \frac{\langle -1|-1\rangle}{\sqrt{2}}\right) & \frac{\langle -1|0\rangle}{\sqrt{2}} \end{pmatrix} \end{array} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{+1}{\sqrt{2}} & 0 \end{pmatrix}$$

$$\langle I_x \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & +1 & 0 \end{pmatrix}$$

I-11)

$I_y \Rightarrow \dfrac{I_+ - I_-}{2i}$

$$I_y|1\rangle = \frac{I_+|1\rangle - I_-|1\rangle}{2i} = \left(-\frac{\sqrt{2}}{2i}|0\rangle\right) = \left(\frac{-|0\rangle}{\sqrt{2}i}\right)$$

$$I_y|0\rangle = \frac{I_+|0\rangle - I_-|0\rangle}{2i} = \left(\frac{\sqrt{2}|1\rangle - \sqrt{2}|-1\rangle}{2i}\right) = \left(\frac{|1\rangle - |-1\rangle}{\sqrt{2}i}\right)$$

$$I_y|-1\rangle = \frac{I_+|-1\rangle - I_-|-1\rangle}{2i} = \frac{\sqrt{2}|0\rangle}{2i} = \frac{|0\rangle}{\sqrt{2}i}$$

$$\begin{pmatrix} \langle 1|I_y|1\rangle & \langle 1|I_y|0\rangle & \langle 1|I_y|-1\rangle \\ \langle 0|I_y|1\rangle & \langle 0|I_y|0\rangle & \langle 0|I_y|-1\rangle \\ \langle -1|I_y|1\rangle & \langle -1|I_y|0\rangle & \langle -1|I_y|-1\rangle \end{pmatrix} = \frac{1}{\sqrt{2}i}\begin{pmatrix} -\langle 1|0\rangle & (\langle 1|1\rangle - \langle 1|-1\rangle) & \langle 1|0\rangle \\ -\langle 0|0\rangle & \langle 0|1\rangle - \langle 0|-1\rangle & \langle 0|0\rangle \\ -\langle -1|0\rangle & \langle -1|1\rangle - \langle -1|-1\rangle & \langle -1|0\rangle \end{pmatrix}$$

$\dfrac{\langle 1|0\rangle}{\sqrt{2}i}$ $\dfrac{(\langle 1|1\rangle - \langle 1|-1\rangle)}{\sqrt{2}i}$ $\dfrac{\langle 1|0\rangle}{\sqrt{2}i}$

$\cancel{\left(\dfrac{\langle 0|1\rangle - \langle 0|-1\rangle}{\sqrt{2}i}\right)}$

$$= \frac{1}{i\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\boxed{\langle I_y\rangle = \frac{1}{i\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}} = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

I-1) $I^2 = I_x^2 + I_y^2 + I_z^2$

$$I_x^2 = \frac{1}{2}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix} = \begin{pmatrix}1&0&1\\0&2&0\\1&0&1\end{pmatrix}\frac{1}{2}$$

$$I_y^2 = -\frac{1}{2}\begin{pmatrix}0&-1&0\\1&0&-1\\0&1&0\end{pmatrix}\begin{pmatrix}0&-1&0\\1&0&-1\\0&1&0\end{pmatrix} = \begin{pmatrix}-1&0&1\\0&-2&0\\1&0&-1\end{pmatrix}\frac{-1}{2} = \frac{1}{2}\begin{pmatrix}1&0&-1\\0&2&0\\-1&0&1\end{pmatrix}$$

$$I_z^2 = \hbar^2\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}\hbar^2$$

$$I_x^2 + I_y^2 = \frac{1}{2}\left[\begin{pmatrix}1&0&1\\0&2&0\\1&0&1\end{pmatrix} + \begin{pmatrix}1&0&-1\\0&2&0\\-1&0&1\end{pmatrix}\right] = \frac{1}{2}\begin{bmatrix}2&0&0\\0&4&0\\0&0&2\end{bmatrix} = \cancel{\hbar} = \begin{pmatrix}1&0&0\\0&2&0\\0&0&1\end{pmatrix}$$

$$I_z^2 + (I_x^2 + I_y^2) = \hbar^2\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix} + \hbar^2\begin{pmatrix}1&0&0\\0&2&0\\0&0&1\end{pmatrix}$$

$$I_z^2 + I_x^2 + I_y^2 = \hbar^2\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix} = 2\hbar^2\,\mathbb{1}$$

I-12) Generalize the expressions $\langle M_{x,y,z}(t)\rangle$ For $S=1$

$\boxed{\text{Hilbert space here}}$

$$\langle M_x(t)\rangle = \gamma\hbar S_z \qquad S_z = \begin{array}{c} \begin{array}{ccc} |1\rangle & |0\rangle & |-1\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{array}$$

Note

$I_z|1\rangle = \hbar$

$I_z|0\rangle = 0$

$I_z|-1\rangle = -\hbar$

$$|\psi(t)\rangle = a_1^0|1\rangle + a_0^0|0\rangle + a_{-1}^0|-1\rangle$$

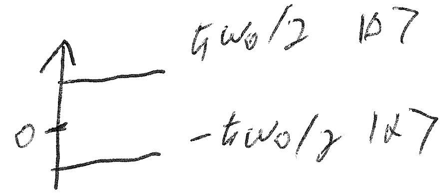
$m_s = \{-s, \ldots s\}$

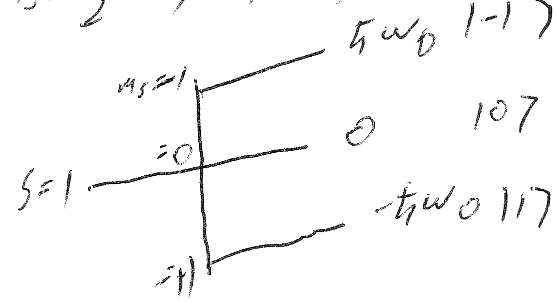
$$E = -\vec{\mu}\cdot B_0 = -S_z\hbar\gamma B_0 = -m_s\hbar\gamma B_0$$

$m_s = \{-1, 0, 1\}$

$$E_{m_s} = -m_s\hbar\omega_0$$

$s=1$





$s=\frac{1}{2}$ $(-1/2, 1/2)$

$|\beta\rangle$ $|\alpha\rangle$

$$|\psi(t)\rangle = a_1|1\rangle + a_0|0\rangle + a_{-1}|-1\rangle$$

$$= ae^{i\alpha}|1\rangle + be^{ib}|0\rangle + ge^{i\gamma}|-1\rangle$$

$a_1 = ae^{i\alpha}$

$a_0 = be^{ib}$

$a_{-1} = ge^{i\gamma}$

$$\boxed{\text{Note: } c_j(t) = c_j^0 e^{-i\left(\frac{E_j}{\hbar}\right)t} = c_j^0 e^{-i\omega_j t}}$$

$\frac{E_j}{\hbar} = \omega_j$ = Bohr Frequency

$$= c_j^0 e^{i(-\omega_j t)}$$

$c_j(t) = c_j^0 e^{-i\omega_0 t} = c_j^0 e^{i(-\omega_j t)}$

$$\boxed{|\Psi(t)\rangle = a_0 e^{i(+\omega_0 t+\alpha)}|1\rangle + b_0 e^{i(\beta)}|0\rangle + g_0 e^{i(-\omega_0 t+\gamma)}|-1\rangle}$$

$S=1$ Time dep wave func (as defined by $\hat{H}$)

Let $A = \omega_0 t + \alpha$
$B = \beta$
$G = -\omega_0 t + \gamma$

~~$|\Psi(t)\rangle = a_0 e_1^{iA} + b e^{iB} + g e^{iG}$~~

$|\Psi(t)\rangle = a_0 e^{iA}|1\rangle + b e^{iB}|0\rangle + g e^{iG}|-1\rangle$

$\langle M\rangle = \gamma\hbar\langle I_{x,y,z}\rangle$

Note $S=1 \Rightarrow$

$$I_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$I_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$I_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

From previous problem J-11

we have everything now $|\Psi(t)\rangle$ & $\langle I_z\rangle$ $\boxed{\text{SPIN}=1}$

$\langle S_z\rangle = \gamma\hbar\langle I_z\rangle \Rightarrow$

$$\gamma\hbar^2\left(a_0 e^{-iA}\ \ b e^{-iB}\ \ g e^{-iG}\right)\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}\begin{pmatrix}a e^{iA}\\ b e^{iB}\\ g e^{iG}\end{pmatrix}$$

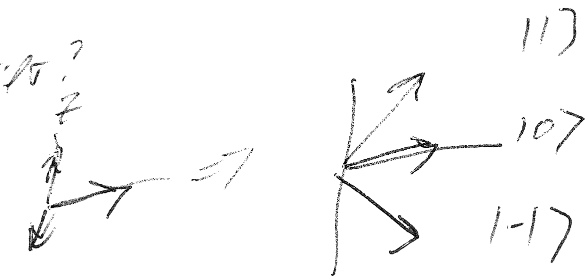
$$=\left(a e^{-iA}\ b e^{-iB}\ g e^{-iG}\right)\begin{pmatrix}a e^{iA}\\ 0\\ -g e^{iG}\end{pmatrix} = a e^{-iA}\cdot a e^{iA} - g e^{-iG} g e^{iG}$$

$$=\left(a^2-g^2\right)\gamma\hbar^2$$

$$\boxed{\langle S_z\rangle = \gamma\hbar^2\left(a^2-g^2\right)} = \cancel{\gamma\hbar^2\left(a_{1/2}^2-0\right.}$$

$$=\gamma\hbar^2\left(a_1^2-a_{-1}^2\right)$$

a & g describe tilt?



$$\langle S_x \rangle = \gamma\hbar \langle I_x \rangle = \frac{\gamma\hbar}{\sqrt{2}} \left(a_0 e^{-iA} \quad b e^{-iB} \quad c e^{-iG} \right) \begin{pmatrix} 0 & 1 & 0 \\ \phi & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a e^{+iA} \\ b e^{+iB} \\ g e^{+iG} \end{pmatrix}$$

$$= \left(a e^{-iA} \quad b e^{-iB} \quad g e^{-iG} \right) \begin{pmatrix} b e^{iB} \\ a e^{iA} + g e^{iG} \\ b e^{iB} \end{pmatrix}$$

$$= a e^{-iA} b e^{iB} + b e^{-iB} \left(a e^{iA} + g e^{iG} \right) + g e^{-iG} b e^{iB}$$

$$= \cancel{a e^{-iA}} b e = ab e^{i(B-A)} + ab e^{i(A-B)} + bg e^{i(G-B)} + bg e^{i(B-G)}$$

$$= ab \left(e^{-i(A-B)} + e^{i(A-B)} \right) + bg \left(e^{-i(B-G)} + e^{i(B-G)} \right)$$

$$= ab \left[c(A-B) - i s(A-B) + c(A-B) + i s(A-B) \right] + bg \left[c(B-G) - i s(B-G) + c(B-G) + i s(B-G) \right]$$

$$= ab \left[2c(A-B) \right] + bg \left[2c(B-G) \right]$$

$\phi = \gamma - B$

$\Delta = B - \gamma$

$a_1 \Rightarrow A = \omega t + \alpha \qquad A - B = \omega t + (\alpha - \beta)$

$a_0 \Rightarrow B = \beta \qquad B - G = +\omega t + (\beta - \gamma)$

$a_{-1} \Rightarrow G = -\omega t + \gamma$

$$\langle S_x \rangle = \frac{a_1 a_0 \gamma\hbar}{\sqrt{2}} \left[2\cos(\omega t + \alpha - \beta) \right] + \frac{a_0 a_{-1} \gamma\hbar}{\sqrt{2}} \left[2\cos(\omega t + \beta - \gamma) \right]$$

$$\boxed{\langle S_x \rangle = \sqrt{2}\,\gamma\hbar\, a_1 a_0 \cos(\omega t + \alpha - \beta) + \sqrt{2}\,\gamma\hbar\, a_0 a_{-1} \cos(\omega t + \beta - \gamma)}$$

$$\langle M_y \rangle = \gamma \hbar \langle S_y \rangle \qquad S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\langle M_y \rangle = \frac{\gamma \hbar i}{\sqrt{2}} \begin{pmatrix} ae^{-iA} & be^{-iB} & ge^{-iG} \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} ae^{iA} \\ be^{iB} \\ ge^{iG} \end{pmatrix}$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \begin{pmatrix} ae^{-iA} & be^{-iB} & ge^{-iG} \end{pmatrix} \begin{pmatrix} -be^{iB} \\ ae^{iA} - ge^{iG} \\ be^{iB} \end{pmatrix}$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[-abe^{-iA}e^{iB} + be^{-iB}(ae^{iA} - ge^{iG}) + bge^{-iG}e^{iG} \right.$$
$$\left. -bge^{i(G-B)} + bge^{i(B-G)} \right]$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[-abe^{i(B-A)} + abe^{i(A-B)} - bge^{i(G-B)} \right.$$
$$\left. + bg\left(e^{i(B-G)} - e^{-i(B-G)}\right) \right)$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[ab\left(-e^{-i(A-B)} + e^{i(A-B)}\right) + bg \begin{bmatrix} c(B-G) + is(B-G) \\ -c(B-G) + is(B-G) \end{bmatrix} \right)$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[ab \begin{bmatrix} -c(A-B) + is(A-B) \\ c(A-B) + is(A-B) \end{bmatrix} + bg \right.$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[ab\, 2i \sin(A-B) + bg\, 2i \sin(B-G) \right]$$

$$= \frac{\gamma \hbar i}{\sqrt{2}} \left[ab \sin(\omega t + \alpha - B) + bg \sin(\omega t + \beta - \gamma) \right]$$

$$= -\gamma \hbar \sqrt{2} \left[ab \sin(\omega t + \alpha - B) + bg \sin(\omega t + \beta - \gamma) \right]$$

$$\boxed{\langle M_y(t) \rangle = -\gamma \hbar \sqrt{2} \left[a_1 a_0 \sin(\omega t + \alpha - B) + a_0 a_{-1} \sin(\omega t + \beta - \gamma) \right]}$$

* terms add in sin funcs w/ coeffs w increasing s

I-12) Review:

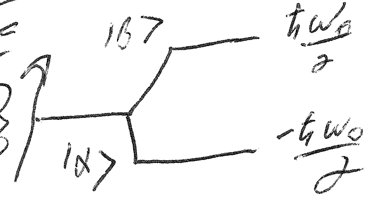
$$H = -\vec{\mu}\cdot\vec{B}$$

$$\mu = \gamma \vec{S} \qquad \vec{B} = b_0 \cdot \hat{z}$$

$$\boxed{H = -\gamma b_0 \cdot S_z}$$

H1

$$H = \{|\alpha\rangle, |\beta\rangle\}$$



$\cancel{\Delta E = \hbar\omega_0}$

$$\boxed{\Delta E = \hbar\omega_0 = h\nu}$$

*Bohr Frequency describes time evolution of the spin

initially: $|\Psi\rangle = a^0_{1/2}|\alpha\rangle + a^0_{-1/2}|\beta\rangle$

$$a^0_{1/2} = a e^{i\alpha}$$
$$a^0_{-1/2} = b e^{i\beta}$$
} The phases

where $a^2 + b^2 = 1$
& $a, b \in \mathbb{R}$ (just real) >0

a & b are scalars describing how much α & β in Ψ

$$\Rightarrow |\Psi(t)\rangle = a e^{i\left(\frac{\omega_0 t}{2} + \alpha\right)}|\alpha\rangle + b e^{i\left(-\frac{\omega_0 t}{2} + \beta\right)}|\beta\rangle$$

With $|\Psi(t)\rangle$ we can now evaluate the time dependence of the magnetization

$$\boxed{\langle \vec{M} \rangle = \gamma\hbar \begin{matrix} \langle S_x \rangle \\ \langle S_y \rangle \\ \langle S_z \rangle \end{matrix}}$$

N = # of spin pairs = 2 (for α & β)

$$\langle M_x(t)\rangle = abN\cos(\omega_0 t + \phi)\cdot(\gamma\hbar)$$
$$\langle M_y(t)\rangle = -abN\sin(\omega_0 t + \phi)\cdot(\gamma\hbar)$$
$$\langle M_z(t)\rangle = N(a^2 - b^2)\left(\frac{\gamma\hbar}{2}\right)$$

Gives magnetic moment for a particular particle

$\langle M_x(t)\rangle = \gamma\hbar\langle S_x\rangle \qquad S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\boxed{|\Psi(t)\rangle = ae^{i\left(\frac{w_0 t}{2}+\alpha\right)}|\alpha\rangle + be^{i\left(-\frac{w_0 t}{2}+\beta\right)}|\beta\rangle}$$

Let $A = \frac{w_0 t}{2} + \alpha$ & $B = \frac{-w_0 t}{2} + \beta$

$|\Psi(t)\rangle = ae^{iA}|\alpha\rangle + be^{iB}|\beta\rangle$

$\langle M_x(t)\rangle = \langle\Psi(t)|S_x|\Psi(t)\rangle\gamma\hbar$

$= (ae^{-iA} + be^{-iB})\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} ae^{iA} \\ be^{iB} \end{pmatrix}\gamma\hbar$

$= (ae^{-iA}\ be^{-iB})\begin{pmatrix} be^{iB} \\ ae^{iA} \end{pmatrix}\gamma\hbar = (abe^{-iA}e^{iB} + abe\,e^{-iB}e^{iA})\gamma\hbar$

$= \gamma ab\hbar\left[e^{i(A-B)} + e^{i(B-A)}\right]$

$A-B = \frac{w_0 t}{2} + \alpha - \left(-\frac{w_0}{t} + \beta\right) \qquad B-A = -\frac{w_0 t}{2} + \beta - \frac{w_0 t}{2} - \alpha \qquad \phi = \alpha - \beta$

$= \frac{w_0 t}{\cancel{2}} + \phi \qquad = -\frac{w_0 t}{\cancel{2}} - \phi$

Let ~~$u = \frac{w_0 t}{2} + \phi$~~ $u = w_0 t + \phi$

$\langle M_x(t)\rangle = \gamma\hbar ab\left[e^{iu} + e^{i(-u)}\right]$

$(e^{iu} + e^{-iu}) =$

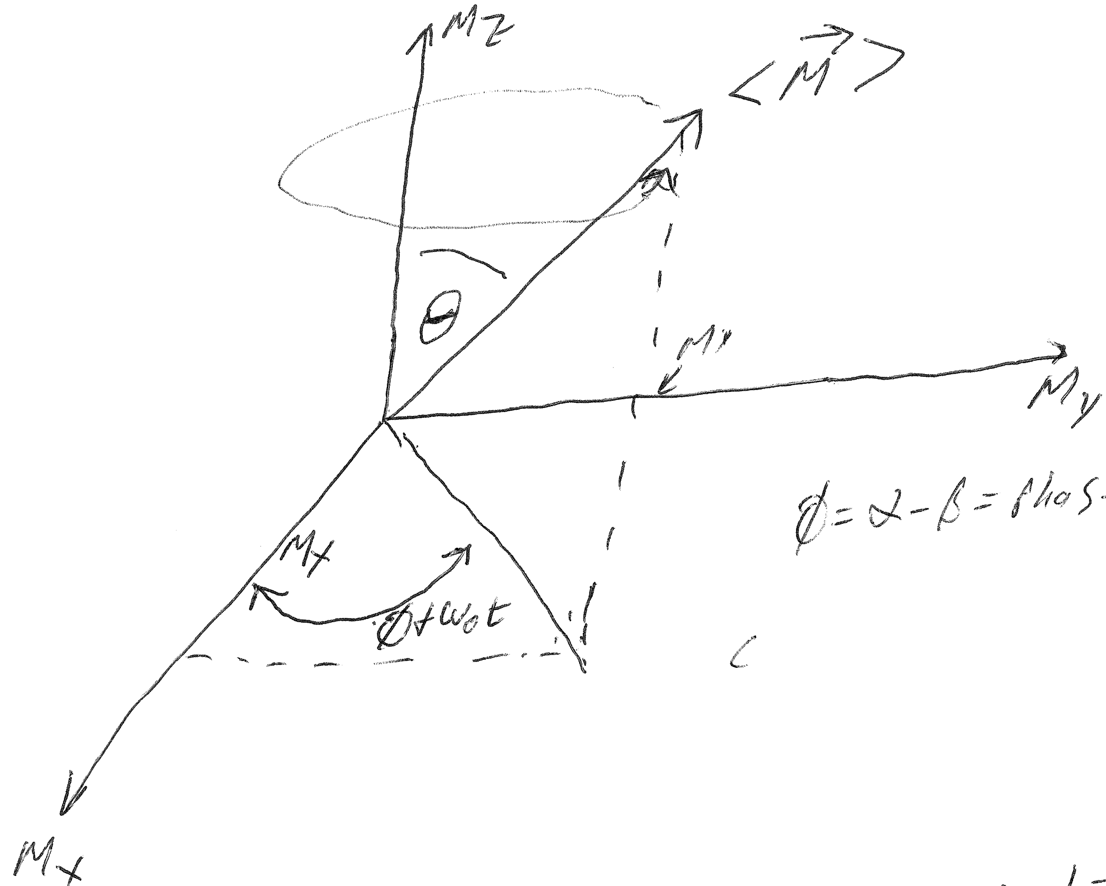
$= \gamma\hbar ab\left[e^{iu} + e^{-iu}\right]$

$= \gamma\hbar ab\left[c(u) + is(u) + c(u) - is(u)\right]$

$= \gamma\hbar ab\left[2 \times c\left(\frac{w_0 t}{\cancel{2}} + \phi\right)\right]$

$$\boxed{\langle M_x(t)\rangle = 2ab\gamma\hbar\cos(w_0 t + \phi)} \qquad \boxed{\langle M_x(t)\rangle = 2ab\gamma\hbar\cos(w_0 t + \phi)}$$

Our vector space



$\phi = \alpha - \beta = $ phase

$$\phi = \alpha - \beta$$

$$\cos(\theta) = 2a^2 - 1 = \begin{cases} 1 \to a = 1,\ b = 0 \\ 0 \to a = b = \frac{1}{2} \\ -1 \to a = 0,\ b = 1 \end{cases}$$

$a^2 + b^2 = 1$ remember

Find these: $\Psi(t) = \cos\left(\frac{\theta}{2}\right) e^{i\left(\frac{\phi}{2} + \omega_0 t\right)} |\alpha\rangle + \sin\left(\frac{\theta}{2}\right) e^{-i\left(\frac{\phi}{2} + \omega_0 t\right)} |\beta\rangle$

$\langle M_z(t)\rangle = \gamma\hbar\langle S_z\rangle$ $\qquad I_z = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$|\psi(t)\rangle = a e^{i(\frac{\omega_0 t}{2}+\alpha)}|\alpha\rangle + b e^{i(-\frac{\omega_0 t}{2}+\beta)}|\beta\rangle$

Let $A = \frac{\omega_0 t}{2}+\alpha$ & $B = -\frac{\omega_0 t}{2}+\beta$

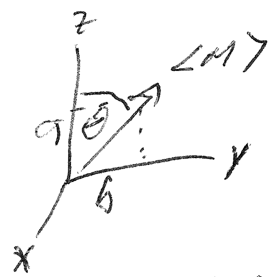
$|\psi(t)\rangle = a e^{iA}|\alpha\rangle + b e^{iB}|\beta\rangle$

$\langle M_z(t)\rangle = \frac{\gamma\hbar}{2}\begin{pmatrix}a e^{-iA} & b e^{-iB}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}a e^{iA}\\ b e^{iB}\end{pmatrix}$

$= \frac{\gamma\hbar}{2}\begin{pmatrix}a e^{-iA} & b e^{-iB}\end{pmatrix}\begin{pmatrix}a e^{iA}\\ -b e^{iB}\end{pmatrix}$

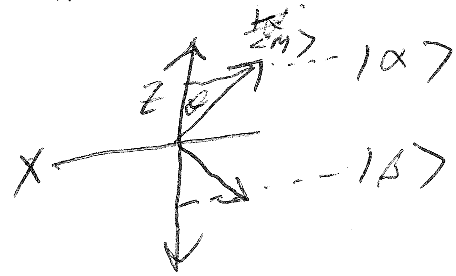
$= \frac{\gamma\hbar}{2}\left[a^2 e^{-iA}e^{iA} - b^2 e^{-iB}e^{iB}\right]$

$\therefore \boxed{\langle M_z(t)\rangle = \frac{\gamma\hbar}{2}\left[a^2-b^2\right]}$



$a^2+b^2=1$ $\qquad c(\theta) = \langle M_z\rangle$

$c(\theta) = 2a^2-1$



$c(\theta) = 2a^2-1 \begin{cases} 1, & a=1, b=0\\ 0, & a=b=1/2\\ -1, & a=0, b=1\end{cases}$

I-13) Find the ~~evolution~~ UNITARY operators; ok too actually!

$$U_j(\phi_j) = e^{i\phi_j I_j} \quad (j = x, y, z)$$

---

use: 1) Pauli matrices $I_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$; $S_z = \frac{2}{\hbar} \cdot I_z \Rightarrow \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$I_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ $\quad S_x = \frac{2}{\hbar} I_x \Rightarrow \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$I_y = \frac{i\hbar}{2}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ $\quad S_y = \frac{2}{\hbar} I_x \Rightarrow \sigma_x = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$

$\sigma_y = 2 I_z \quad \sigma_x = 2 I_x$

$\sigma_z = 2 I_z$

2) use Taylor expansion

$$e^x = 1 + \frac{(x)}{1!} + \frac{(x)^2}{2!} + \frac{(x^3)}{3!} + \frac{(x)^4}{4!}$$

3) use IDENTITY

$$e^{iu} = \left(1 - \frac{u^2}{2!} + \frac{u^4}{4!} + \dots\right) + i\left(\frac{u}{1!} - \frac{u^3}{3!} + \frac{u^5}{5!} + \dots\right)$$

$$= \cos(u) + i \sin(u)$$

---

4) use $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$\sigma_z^2 = \mathbb{1} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$\sigma_z^3 = \mathbb{1}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \sigma_z$

$\vdots$

$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$\sigma_x^2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \mathbb{1}$

$\sigma_x^3 = \sigma_x$

$\sigma_x^4 = \mathbb{1}$

$\sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = i\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

$\sigma_y^2 = i^2\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = -1\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix} = \mathbb{1}$

$\sigma_y^2 = \mathbb{1}$

$\sigma_y^3 = \sigma_y$

$\sigma_y^4 = \mathbb{1}$

Find The Matrix expressions FOR $U_z e^{i I_z \phi_z}$ | * KIND OF combined $I-13, 14$ here

$$\boxed{\#B}\; e^{i\phi_z I_z} = e^{i\left(\frac{\phi_z}{2}\right)2I_z} = e^{i\alpha_z\sigma_z}$$

$$\boxed{\begin{aligned} 2I_z &= \sigma_z \\ \alpha_z &= \left(\frac{\phi_z}{2}\right) \end{aligned}}$$

Don't Forget

$$e^{i\alpha_z\sigma_z} = 1 + \frac{i\alpha_z\sigma_z}{1} + \frac{i^2\alpha_z^2\sigma_z^2}{2} + \frac{(i\sigma_z\alpha_z)^3}{3!} + \frac{(i\alpha_z\sigma_z)^4}{4!}$$

$$= \left(1 - \frac{\alpha_z^2\sigma_z^2}{2!} + \frac{\alpha_z^4\sigma_z^4}{4!} + \dots\right) + i\left(\frac{\alpha_z\sigma_z}{1} - \frac{\alpha_z^3\sigma_z^3}{3!} + \frac{\alpha_z^5\sigma_z^5}{5!}\right)$$

$$\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad \sigma_z^2 = \mathbb{1} \quad \sigma_z^4 = \mathbb{1}$$

$$\sigma_z^3 = \sigma_z \quad \sigma_z^5 = \sigma_z$$

$$e^{i\alpha_z\sigma_z} = \cos(\alpha_z \cancel{t}) + i\,\sigma_z \sin(\alpha_z \cancel{t})$$

$$\boxed{e^{i\alpha_z\sigma_z} = \cos(\alpha_z) + i\,\sigma_z \sin(\alpha_z)}$$

$$\alpha_z = \frac{\phi_z}{2}$$

$$\sigma_z = 2I_z$$

$$\boxed{\begin{aligned} \alpha_i &= \frac{\phi_i}{2} \quad \} x, y, z \\ \sigma_i &= 2I_i \end{aligned}}$$

Summary:

e

Summary:
$e^{i\phi_z I_z} = c(\alpha_z) + i\,\sigma_z\, s(\alpha_z)$
$e^{i\phi_x I_x} = c(\alpha_x) + i\,\sigma_x\, s(\alpha_x)$
$e^{i\phi_y I_y} = c(\alpha_y) + i\,\sigma_y\, s(\alpha_y)$

$$e^{i\phi_x I_x} = e^{i\left(\frac{\phi_x}{2}\right)2I_x} = e^{i\alpha_x\sigma_x} \qquad \alpha_x = \left(\frac{\phi_x}{2}\right)$$

$$\boxed{e^{i\alpha_x\sigma_x} = \cos(\alpha_x) + i\sigma_x\sin(\alpha_x)} \qquad \sigma_x = 2I_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$e^{i\phi_y I_y} = e^{i\left(\frac{\phi_y}{2}\right)2I_y} = e^{i\alpha_y\sigma_y} \qquad \alpha_y = \frac{\phi_x}{2}$$

$$\sigma_y = 2I_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$e^{i\alpha_y\sigma_y} = \cos(\alpha_y)\mathbb{1} + i\sin(\alpha_y)\sigma_y$$

$$\boxed{e^{i\phi_x I_x} = \cos(\alpha_y)\mathbb{1} + i\sigma_y\sin(\alpha_y)}$$

---

could expand these like:

$$e^{i\phi_x I_x} = c(\alpha_x) + i\sigma_x\sin(\alpha_x)$$

$$= \begin{pmatrix}1&0\\0&1\end{pmatrix}c(\alpha_x) + i\begin{pmatrix}0&1\\1&0\end{pmatrix}\sin(\alpha_x)$$

$$= \begin{pmatrix}c(\alpha_x)&0\\0&c(\alpha_x)\end{pmatrix} + \begin{pmatrix}0&is(\alpha_x)\\is(\alpha_x)&0\end{pmatrix}$$

$$\boxed{e^{i\phi_x I_x} = \begin{pmatrix}c(\alpha_x)&is(\alpha_x)\\is(\alpha_x)&c(\alpha_x)\end{pmatrix}} = c(\alpha_x) + i\sigma_x\sin(\alpha_x)$$

$$I_z \xrightarrow{U_x} = e^{i\phi_x I_x} I_z e^{-i\phi_x I_x} = e^{i\left(\frac{\phi_x}{2}\right)2I_x}\left(\frac{\sigma_z}{2}\right)e^{-i\left(\frac{\phi_x}{2}\right)2I_x}$$

$\boxed{\text{I-14}}$

$$= e^{i\alpha_x \sigma_x}\left(\frac{\sigma_z}{2}\right)e^{-i\alpha_x \sigma_x}$$

$\sigma_x = 2I_x$
$\sigma_z = 2I_z$
$\alpha_x = \frac{\phi_x}{2}$

$$= \frac{1}{2}\left[C(\alpha_x) + i\sigma_x S(\alpha_x)\right]\sigma_z\left[C(\alpha_x) - i\sigma_x S(\alpha_x)\right]$$

* Type Rotations

* see HMWRK I-14

$I_x \xrightarrow{U_x} I_x$

$\xrightarrow{U_y} I_x \cos(\phi_y) + I_z \sin(\phi_y)$

$\xrightarrow{U_z} I_x \cos(\phi_z) - I_y \sin(\phi_z)$

$I_y \xrightarrow{U_x} I_y \cos(\phi_x) - I_z \sin(\phi_x)$

$\xrightarrow{U_y} I_y$

$\xrightarrow{U_z} I_y \cos(\phi_z) + I_x \sin(\phi_z)$

$I_z \xrightarrow{U_x} I_z \cos(\phi_x) + I_y \sin(\phi_x)$

$\xrightarrow{U_y} I_z \cos(\phi_y) - I_x \sin(\phi_y)$

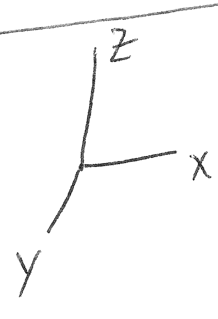
$\xrightarrow{U_z} I_z$

Convention

$\sigma_x\sigma_y = i\sigma_z \quad \sigma_y\sigma_x = -i\sigma_z$

$\sigma_y\sigma_z = i\sigma_x \quad \sigma_z\sigma_y = -i\sigma_x$

$\sigma_z\sigma_x = i\sigma_y \quad \sigma_x\sigma_z = -i\sigma_y$



$$I_y \xrightarrow{u_z} u_z I_z u_z^{-1} = [C(\alpha_z) + i\sigma_z S(\alpha_z)] \left(\frac{\sigma_y}{2}\right) [c(\alpha_z) - i\sigma_z S(\alpha_z)]$$

$$= \left(\frac{1}{2}\right) [c + i\sigma_z S][\sigma_y c - i\sigma_y\sigma_z S]$$

$$= \frac{1}{2} [c^2\sigma_y - i c\sigma_y\sigma_z S + (i\sigma_z S)(\sigma_y c) - (i\sigma_z S)(i\sigma_y\sigma_z S)]$$

$$= \frac{1}{2} [c^2\sigma_y - icS\sigma_y\sigma_z + icS\sigma_z\sigma_y - i^2 S^2 \sigma_z\sigma_y\sigma_z]$$

$x\ y\ \ z \qquad \sigma_z\sigma_y\sigma_z$

~~yx~~ $\qquad = \sigma_z\sigma_x i = \cancel{(-\sigma_y i) i = \sigma_x}$

$y\ z\ \ x$

$z\ x\ \ y \qquad = i^2\sigma_y = -\sigma_y$

$$= \frac{1}{2} [c^2\sigma_y - icS(i\sigma_x) + icS(-i\sigma_x) - i^2 S^2(-\sigma_y)]$$

$$= \frac{1}{2} [c^2\sigma_y - i^2cS\sigma_x - i^2cS\sigma_x - i^2S^2(-\sigma_y)]$$

$$= \frac{1}{2} [c^2\sigma_y + cS\sigma_x + cS\sigma_x - S^2\sigma_y]$$

$$= \frac{1}{2} [c^2\sigma_y + cS\sigma_x$$

$$= \frac{1}{2} [\sigma_y[c^2(\alpha_z) - S^2(\alpha_z)] + 2c(\alpha_z)S(\alpha_z)\sigma_x]$$

$$c(\alpha)S(\alpha) = \frac{S(2\alpha)}{2} \qquad c^2(\alpha) - S^2(\alpha) = C(2\alpha)$$

$$= \frac{1}{2}\left[\sigma_y C(2\alpha_z) + 2\frac{\sigma_x S(\alpha_z)}{2}\right] = \frac{1}{2}[\sigma_y C(2\alpha_z) + \sigma_x S(2\alpha_z)]$$

$\sigma_y = 2I_x$

$\sigma_x = 2I_x$

$$\frac{1}{2}\left[2I_x C\left(\frac{2\phi_z}{2}\right) + 2I_y S\left(\frac{2\phi_z}{2}\right)\right]$$

$$I_y \xrightarrow{u_z} I_x\cos(\phi_z) + I_y S(\phi_z)$$

$$I_y \xrightarrow{u_x} e^{i\phi_x I_x} I_y e^{-i\phi_x I_x} = e^{i\left(\frac{\phi_x}{2}\right)2I_x} \left(\frac{\sigma_y}{2}\right) e^{-i\left(\frac{\phi_x}{2}\right)2I_x}$$

$$\boxed{\begin{array}{ll} \sigma_x = 2I_x & \alpha_x = \left(\frac{\phi_x}{2}\right) \\ \sigma_y = 2I_y & \end{array}}$$

$$= \frac{1}{2} e^{i\alpha_x\sigma_x} \sigma_y e^{-i\alpha_x\sigma_x}$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\sigma_y\left[c - i\sigma_x s\right]$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\left[\sigma_y c - \sigma_y i\sigma_x s\right]$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\left[c\sigma_y - is\sigma_y\sigma_x\right]$$

$$= \frac{1}{2}\left[cc\sigma_y - ics\sigma_y\sigma_x + (i\sigma_x s)(c\sigma_y) - (i\sigma_x s)(is\sigma_y\sigma_x)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_y - ics\sigma_y\sigma_x + ics\,\sigma_x\sigma_y - i^2s^2\,\sigma_x\sigma_y\sigma_x\right]$$

$$\sigma_x\sigma_y = i\sigma_z \qquad \sigma_y\sigma_x = (-i\sigma_z)$$

$$\begin{array}{l} xyz \\ yzx \\ zxy \end{array}$$

$$\sigma_x\sigma_y\sigma_x = \sigma_x(-i\sigma_z) = -i(\sigma_x\sigma_z)$$
$$-\sigma_y = i(-i\sigma_y)$$

$$= \frac{1}{2}\left[c^2\sigma_y - ics(-i\sigma_z) + ics(i\sigma_z) - i^2s^2(-\sigma_y)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_y + i^2cs\sigma_z + i^2cs\sigma_z + i^2s^2\sigma_y\right]$$

$$= \frac{1}{2}\left[c^2\sigma_y - cs\sigma_z - cs\sigma_z - s^2\sigma_y\right]$$

$$c(\alpha)s(\alpha) = \frac{s(2\alpha)}{2} \qquad c^2(\alpha) - s^2(\alpha) = c(2\alpha)$$

$$= \frac{1}{2}\left[\sigma_y c(2\alpha_x) - 2\left(\frac{s(2\alpha_x)}{2}\right)\sigma_z\right]$$

$$\begin{array}{l} \sigma_z = 2I_z \\ \sigma_y = 2I_y \\ \alpha_x = \frac{\phi_x}{2} \end{array}$$

$$= \frac{1}{2}\left[\sigma_y c(2\alpha_x) - s(2\alpha_x)\sigma_z\right]$$

$$= \frac{1}{2}\left[\sigma_y \cos(2\alpha_x) - 2I_z \sin\left(\frac{2\phi_x}{2}\right)\right]$$

$$= \frac{1}{2}\left[2I_y \cos\left(\frac{2\phi_x}{2}\right) - \ldots\right.$$

$$\Rightarrow I_y \xrightarrow{u_x} \boxed{I_y\cos(\phi_x) - I_z\sin(\phi_x)}$$

$$I_x \xrightarrow{u_y(\phi_y)} e^{i\phi_y I_y} I_x e^{-i\phi_y I_y}$$

$\sigma_x = 2I_x \quad \therefore I_x = \frac{\sigma_x}{2}$

$\sigma_y = 2I_y$

$\alpha_y = \frac{\phi_y}{2}$

$$= e^{i\left(\frac{\phi_y}{2}\right)2I_y}\left(\frac{\sigma_x}{2}\right)e^{-i\left(\frac{\phi_y}{2}\right)2I_y}$$

$$= e^{i\alpha_y\sigma_y}\left(\frac{\sigma_x}{2}\right)e^{-i\alpha_y\sigma_y}$$

$e^{i\alpha_y\sigma_y} = U_y = C(\alpha_y) + i\sigma_y S(\alpha_y)$

$$= (C + i\sigma_y S)\left(\frac{\sigma_x}{2}\right)(C - i\sigma_y S)$$

$$= \frac{1}{2}(C + i\sigma_y S)\left[C\sigma_x - i\sigma_x\sigma_y S\right]$$

$$= \frac{1}{2}\left[C \cdot C\sigma_x - iC\sigma_x\sigma_y S + (i\sigma_y S)(C\sigma_x) - i^2\sigma_y S\sigma_x\sigma_y S\right.$$

$$= \frac{1}{2}\left[C^2\sigma_x - iCS\sigma_x\sigma_y + iCS\sigma_y\sigma_x - i^2S^2\sigma_y\sigma_x\sigma_y\right.$$

x y z
y z x
z x y

$\sigma_x\sigma_y = i\sigma_z$

$yx = -i\sigma_z$

$\sigma_y\sigma_x\sigma_y$

$\sigma_y(i\sigma_z)$

$i(i\sigma_x) = -\sigma_x$

$$= \frac{1}{2}\left[C^2\sigma_x - iCS(i\sigma_z) + iCS(-i\sigma_z) - i^2S^2(-\sigma_x)\right]$$

$$= \frac{1}{2}\left[C^2\sigma_x - i^2CS\sigma_z - i^2CS\sigma_z + i^2S^2\sigma_x\right]$$

$$= \frac{1}{2}\left[C^2\sigma_x + CS\sigma_z + CS\sigma_z - S^2\sigma_x\right)$$

$$= \frac{1}{2}\left[C^2\sigma_x + 2CS\sigma_z\right]$$

$$\boxed{\begin{array}{l} C(\alpha)S(\alpha) = \frac{S(2\alpha)}{2} \\ C^2(\alpha) - S^2(\alpha) = C(2\alpha) \end{array}}$$

$$= \frac{1}{2}\left[\sigma_x(C^2 - S^2) + 2CS\sigma_z\right]$$

$$= \frac{1}{2}\left[\sigma_x\cos(2\alpha_y) + 2\left(\frac{\sin(2\alpha_y)}{2}\right)\sigma_z\right]$$

$\alpha_y = \frac{\phi_y}{2}$

$\sigma_x = 2I_x$

$\sigma_z = 2I_z$

$$= \frac{1}{2}\left[\sigma_x\cos(2\alpha_y) + \sin(2\alpha_y)\sigma_z\right]$$

$$= \frac{1}{2}\left[2I_x\cos\left(2\frac{\phi_y}{2}\right) + 2I_z\sin\left(\frac{2\phi_y}{2}\right)\right]$$

$$\boxed{I_x \xrightarrow{u_y} = I_x\cos(\phi_y) + I_z\sin(\phi_y)}$$

$$I_x \xrightarrow{u_z(\phi_z)} e^{i\phi_x I_z} I_x e^{-i\phi_x I_z} = \qquad \sigma_x = 2 I_x$$

$$= \left[c(\alpha_z) + i\sigma_z s(\alpha)\right]\left(\frac{\sigma_x}{2}\right)\left[c(\alpha_z) - i\,\sigma_z\, s(\alpha_z)\right]$$

$$= (c + i s\sigma_z)\frac{1}{2}\left[c\sigma_x - i\,\sigma_x\sigma_z s\right]$$

$$= \frac{1}{2}\left[c(c\sigma_x) + c(-i\sigma_x\sigma_z s) + is\sigma_z(c\sigma_x) + is\sigma_z(-i\sigma_x\sigma_z s)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_x - ics\,\sigma_x\sigma_z + ics\,\sigma_z\sigma_x - i^2 s^2\sigma_z\sigma_x\sigma_z\right]$$

$\sigma_x\sigma_y = i\sigma_z$ x y z   else $(-i\sigma_z)$

y z x

z x y

$$= \frac{1}{2}\left[c^2\sigma_x - ics(-i\sigma_y) + ics(+i\sigma_y) - i^2 s^2(-\sigma_x)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_x - cs\sigma_y + i^2 cs\sigma_y - s^2\sigma_y\right]$$

$$= \frac{1}{2}\left[c^2\sigma_x - s^2\sigma_y\right] - 2cs\sigma_y$$

$$\boxed{\begin{aligned} c(\alpha)s(\beta) &= \frac{s(\alpha+\beta)+s(\alpha-\beta)}{2} \\ c(\alpha)c(\beta) - s(\alpha)s(\beta) &= c(\alpha+\beta) \end{aligned}}$$

$$= \frac{1}{2}\left[\cancel{c(\alpha_z)c(\alpha_z)}\right.$$

$$= \frac{1}{2}\left[c^2(\alpha_z)\sigma_x - s^2(\alpha_z)\sigma_x\right] - 2c(\alpha_z)s(\alpha_z)\sigma_y$$

$$= \frac{1}{2}\left[\sigma_x c(2\alpha_z) - 2\left[\frac{s(2\alpha_z) + \overset{\text{zero}}{s(\phi)}}{2}\right]\sigma_y\right. \qquad s(0)=0$$

$$\alpha_z = \frac{\phi_z}{2}$$

$$= \frac{1}{2}\left[\sigma_x c(2\alpha_z) - \sigma_y s(2\alpha_z)\right)$$

$$I_x \xrightarrow{u_x(t)} \frac{1}{2}\left[\sigma_x \cos(\phi_z) - \sigma_y \sin(\phi_z)\right]$$

$$= \frac{1}{2}\left[2I_x \cos(\phi_z) - 2I_y \sin(\phi_z)\right]$$

$$\boxed{I_x(\phi_z) \overset{u_z}{=} I_x \cos(\phi_z) - I_y \sin(\phi_z)} \quad \checkmark$$

$$u_z \xrightarrow{I_z} u_z I_z u_z$$

$$= \left[c(\alpha_z) + i\sigma_z s(\alpha_z)\right]\left(\frac{\sigma_z}{2}\right)\left[c(\alpha_z) - i s(\alpha_z)\sigma_z\right]$$

$$= \frac{1}{2}\left[c + i\sigma_z s\right]\left[c\sigma_z - i s \sigma_z \sigma_z\right)$$

$$= \frac{1}{2}\left[c^2\sigma_z - i c s \sigma_z^2 + i\sigma_z s c \sigma_z - i^2 \sigma_z s s \sigma_z^2\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - i c s \sigma_z^2 + i\sigma_z s c\sigma_z - i^2\sigma_z s^2\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - ics + ics - i^2\sigma_z s^2\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z + s^2\sigma_z\right) = \frac{1}{2}\sigma_z\left[\cancel{c^2+s^2}^{\,1}\right] = \frac{1}{2}\sigma_z$$

$$\sigma_z = 2I_z \quad \therefore \quad I_z \xrightarrow{u_z} I_z$$

$$I_z \xrightarrow{u_x} e^{i\phi_x I_x} I_z e^{-i\phi_x I_x} = e^{i\left(\frac{\phi_x}{2}\right)2I_x}\left(\frac{\sigma_z}{2}\right)e^{-i\left(\frac{\phi_x}{2}\right)2I_x}$$

$$\boxed{\begin{aligned}\sigma_x &= 2I_x\\ \sigma_z &= 2I_z\\ \alpha_{\cancel{x}} &= \left(\frac{\phi_x}{2}\right)\end{aligned}}$$

$$= \frac{1}{2} e^{i\alpha_x\sigma_x}\sigma_z e^{-i\alpha_x\sigma_x}$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\sigma_z\left[c - i\sigma_x s\right]$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\left[\sigma_z c - \sigma_z i\sigma_x s\right]$$

$$= \frac{1}{2}\left[c + i\sigma_x s\right]\left[c\sigma_z - i\sigma_z\sigma_x s\right]$$

$$= \frac{1}{2}\left[c c\sigma_z - c i\sigma_z\sigma_x s + (i\sigma_x s)(c\sigma_z) - (i\sigma_x s)(i\sigma_z\sigma_x s)\right]$$

$$= \frac{1}{2}\left[c c\sigma_z - c i\sigma_z\sigma_x s + i c s\sigma_x\sigma_z - i^2 s^2\sigma_x\sigma_z\sigma_x\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - i c s\sigma_z\sigma_x + i c s\sigma_x\sigma_z - \right.$$

$$\sigma_z\sigma_x = i\sigma_y \qquad xyz$$
$$\sigma_x\sigma_z = -i\sigma_y \qquad yzx$$
$$zxy$$

$$\sigma_x\sigma_z\sigma_x$$
$$\sigma_x(i\sigma_y) = i(\sigma_x\sigma_y)$$
$$= i(i\sigma_z) = -\sigma_z$$

$$= \frac{1}{2}\left[c^2\sigma_z - i c s(i\sigma_y) + i c s(-i\sigma_y) - i^2 s^2(-\sigma_z)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - i^2 c s\sigma_y - i^2 c s\sigma_y + i^2 s^2\sigma_z\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z + c s\sigma_y + c s\sigma_y - s^2\sigma_z\right.$$

$$c^2(\alpha) - s^2(\alpha) = c(2\alpha) \qquad c(\alpha)s(\alpha) = \frac{s(2\alpha)}{2}$$

$$= \frac{1}{2}\left[\sigma_z c(2\alpha_x) + 2\left(\frac{s(2\alpha_x)}{2}\right)\sigma_y\right]$$

$$\sigma_z = 2I_z$$
$$\sigma_x$$

$$= \frac{1}{2}\left[\sigma_z\cos(2\alpha_x) + \sin(2\alpha_x)\sigma_y\right]$$

$$= \frac{1}{2}\left[\sigma_z\cos(2\alpha_x) + 2I_y\sin\left(\frac{2\phi_x}{2}\right)\right]$$

$$= \frac{1}{2}\left[2I_z\cos\left(\frac{2\phi_x}{2}\right) + 2I_y\sin\left(\frac{2\phi_x}{2}\right)\right]$$

$$\boxed{= I_z\cos(\phi_x) + I_y\sin(\phi_x) \Rightarrow I_z \xrightarrow{u_x}}$$

$$I_z \xrightarrow{u_y} e^{i\phi_y I_y} I_z e^{-i\phi_y I_y} = e^{i\left(\frac{\phi_y}{2}\right)2I_y}\left(\frac{\sigma_z}{2}\right)e^{-i\left(\frac{\phi_y}{2}\right)2I_y}$$

$$= \left(\frac{1}{2}\right) e^{i\alpha_y \sigma_y}\sigma_z e^{-i\alpha_y\sigma_y}$$

$$\boxed{\begin{array}{l} \sigma_y = 2I_y \quad \sigma_z = 2I_z \\ \alpha_y = \frac{\phi_y}{2} \end{array}}$$

$$= \frac{1}{2}\left[c + is\sigma_y\right]\sigma_z\left[c - i\sigma_y s_y\right]$$

$$= \frac{1}{2}\left[c + is\sigma_y\right]\left[\sigma_z c - \sigma_z i\sigma_y s\right]$$

$$= \frac{1}{2}\left[c + is\sigma_y\right]\left[c\sigma_z - is\sigma_z\sigma_y\right]$$

$$= \frac{1}{2}\left[cc\sigma_z - ics\sigma_z\sigma_y + (is\sigma_y)(c\sigma_z) - (is\sigma_y)(is\sigma_z\sigma_y))\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - ics\sigma_z\sigma_y + ics\sigma_y\sigma_z - i^2s^2\sigma_y\sigma_z\sigma_y\right]$$

$\sigma_z\sigma_y = -i\sigma_x$

$\sigma_y\sigma_z = i\sigma_x$

$\sigma_x\sigma_y = i\sigma_z$

$y\,z = x$

$z\,x = y$

$\sigma_y\sigma_z\sigma_y$

$= \sigma_y(-i\sigma_x)$

$= -i(\sigma_y\sigma_x) = -i(-i\sigma_z)$

$= -\sigma_z$

$$= \frac{1}{2}\left[c^2\sigma_z - ics(-i\sigma_x) + ics(i\sigma_x) - i^2s^2(-\sigma_z))\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z + i^2cs\sigma_x + i^2cs\sigma_x + i^2s^2\sigma_z)\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - cs\sigma_x - cs\sigma_x - s^2\sigma_z\right]$$

$$= \frac{1}{2}\left[c^2\sigma_z - 2cs\sigma_x\right.$$

$c^2(\alpha) - s^2(\alpha) = c(2\alpha) \qquad c(\alpha)s(\alpha) = \frac{s(2\alpha)}{2}$

$\alpha_y = \frac{\phi_y}{2}$

$$= \frac{1}{2}\left[\sigma_z c(2\alpha_y) - 2\left(\frac{\sin(2\alpha_y)}{2}\right)\sigma_x\right.$$

$\sigma_x = 2I_x$

$\sigma_z = 2I_z$

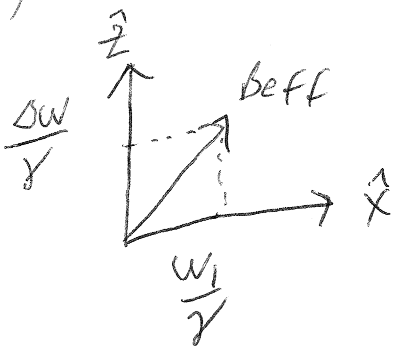
$$= \frac{1}{2}\left[2I_z\cos\left(\frac{2\times\phi_y}{2}\right) - \sin\left(\frac{2\times\phi_y}{2}\right)(2I_x)\right]\right]$$

$$\boxed{= I_z\cos(\phi_y) - I_x\sin(\phi_y) \Rightarrow I_z \xrightarrow{u_y}}$$

I-15) In the presence of b_1 field oscillating at rate ω_1

$$\boxed{B_{eff} = \frac{1}{\gamma}\left(\Delta\omega\,\hat{z} + \omega_1\,\hat{x}\right)} = \frac{b_1}{\gamma}\hat{x} + \left(b_0 - \frac{\omega}{\gamma}\right)\hat{z}$$

i) calculate the precession rate of magnete $\omega = \gamma b_0$
$= \gamma\, b_{eff}$



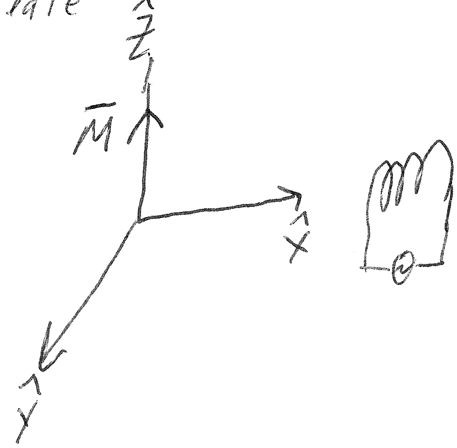
$x^2 + y^2 = z^2$

$$b_{eff} = \sqrt{\left(\frac{\Delta\omega}{\gamma}\right)^2 + \left(\frac{\omega_1}{\gamma}\right)^2}$$

$$\therefore\ \omega = \gamma B_{ef} = \gamma\sqrt{\left(\frac{\Delta\omega}{\gamma}\right)^2 + \left(\frac{\omega_1}{\gamma}\right)^2}$$

$$\boxed{\tan\left(\frac{\Delta\omega}{\omega_1}\right) = \theta\ ?}$$

ii) calculate the trajectory of M if $M(t=0) = (0,0,M_0)$



This: $U_x M_z U_x$ rotation

$$\therefore\ \boxed{M(t) = M_z C(\omega_1 t) + M_y S(\omega_1 t)}$$

Lowles

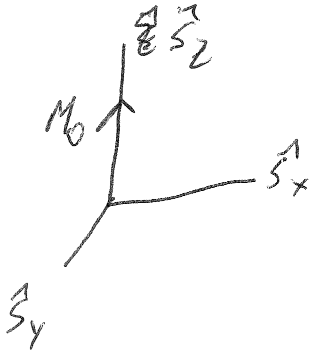
* see I-14 unitary ops

$\omega_0 - \omega = 0$

$$H_r = -\hbar(\omega_0 - \omega)S_z - \hbar\omega_1 S_x$$

$$H_r = -\hbar\omega_1 S_x$$

15-ii) Calculate The Trajectory of M if $M(t=0)=(0,0,M_0)$



$\rho(t=0) = M_0 S_z$

Rotation
Order Matters!
1^{st} - x
2^{nd} - z

Let $\omega_0 - \omega_0 = \Delta$ = offset

$$\boxed{\hat{H}_r = -\hbar(\omega_0-\omega)S_z - \hbar\omega_1 S_x}$$

$$\rho(t) = e\left(-\frac{i\hat{H}t}{\hbar}\right)\rho(t=0)\, e\left(\frac{i\hat{H}t}{\hbar}\right)$$

$$= \cancel{e(i\Delta\omega t) M_0 S_z e(-i\Delta t)}$$

$$= e(i[\Delta\omega t S_z + \omega_1 t S_x])\, M_0 S_z\, e(-i[\Delta\omega t S_z + \omega_1 t S_x])$$

$$= e(i\Delta\omega t S_z) e(i\omega_1 t S_x)\, M_0 S_z\, e(-i\Delta\omega t S_z) e(-i\omega_1 t S_x)$$

Let $\Delta\omega t = \phi_z$ $\quad \omega_1 t = \theta$

$$e(i\phi S_z) e(i\theta S_x)\, M_0 S_z\, e(-i\theta S_x) e(-i\phi S_z)$$

$$M_0\left[U_z(\phi)\, U_x(\theta)\, S_z\, U_x^{-}(\theta)\, U_z^{-}(\phi)\right]$$ — See I-14 Ross unitary ops

$$\rho(t) = M_0\, U_z(\phi)\left[\, S_z\, c(\theta) + S_y\, s(\theta)\right] U_{-z}(\phi)$$

$$P(t) = M_0 \{ U_z(\phi)\, S_z\, C(\theta)\, U_z^{-}(\phi) + U_z(\phi)\, S_x\, S(\theta)\, U_{+z}(\phi) \}$$

$$= M_0 C(\theta) \left[U_z \phi\, S_z\, U_z^{-1}(\phi) \right] + M_0 S(\theta \left[U_z(\phi)\, S_y\, U_{+z}(\phi) \right] \}$$

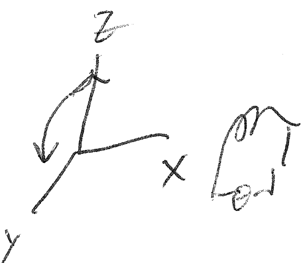
$$P(t) = M_0 C(\theta) S_z + M_0 S(\theta) \left[S_{\underset{y}{\cancel{x}}}\, C(\phi) + S_{\underset{x}{\cancel{x}}}(\phi) \right]$$

$\cancel{\phi}$ $\quad \theta = w_1 t$

$\phi = \Delta w t$

$$\therefore \boxed{P(t) = M_0 \cos(w_1 t) S_z + M_0 \sin(w_1 t) \left[S_{\underset{y}{\cancel{y}}} \cos(\Delta w t) + S_{\underset{x}{\cancel{x}}} \sin(\Delta w t) \right)}$$

$\llcorner$ rotating frame RF pulse w/offset



if $\Delta w = 0$

$$P(t) = M_0 C(w_1 t) S_z + M_0 S(w_1 t) S_{\cancel{x}}$$

$$= M_0 \left[C(w_1 t) S_z + S(w_1 t) S_{\cancel{x}} \right]$$

$$\boxed{P(t) = M_0 \left\{ C(w_1 t) S_z + \cancel{M_0}\, S(w_1 t) \left[S_y C(\Delta w t) + S_x S(\Delta w t) \right] \right\}}$$

I-15) iii) What range of ω_1 (as a function of $\Delta\omega$) are required to bring magnetization from Z into X-Y plane?

$$\rho(t) = M_{0Z}\{C(\omega_1 t) S_Z + S(\omega_1 t)[S_Y C(\Delta\omega t) + S_X S(\Delta\omega t)]$$ Rot about X

$\omega_1 t >> \Delta\omega t$

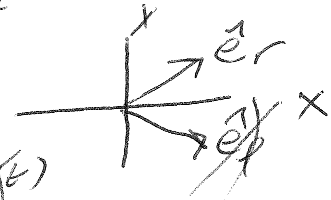
---

iiii) Describe the phase that M_0 originally // to Z - makes with ~~the~~ WRT Y-axis as a $f(\Delta\omega)$ @ const = ω_1

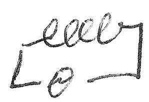
Phase along Y-axis modulates @ $C(\Delta\omega t) * S(\omega_1 t)$

I-15) continued, have classical description of B_{eff} in rotating frame too.

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \left[B_0 \hat{z} + \frac{B_1}{2} \hat{e}_r \right]$$

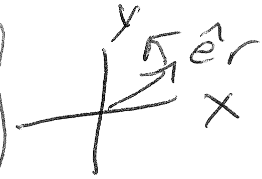


keeping just $\hat{e}_r$ component $c(\omega t) + c(\cancel{-\omega t})$



$\vec{I} = I_0 c(\omega t)$

$B = B_0 c(\omega t)$



Lab frame classic

$$\boxed{\frac{d\vec{M}}{dt} = \gamma \vec{M} \left[B_0 \hat{z} + \frac{B_1}{2} \left[c(\omega t) \hat{x} + s(\omega t) \hat{y} \right] \right]}$$

$\omega = \gamma B$

$c(\omega t)\hat{x} + s(\omega t)\hat{y}$ with $c(0) = 1$, $s(0) = 0$

$c((\omega - \omega)t) = c(0)$

Rotating frame classic

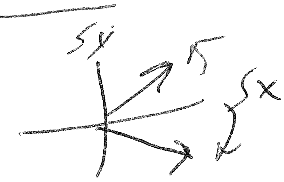
$$\boxed{\frac{\delta M}{\delta t} = \gamma \vec{M} \left[\left(B_0 - \frac{\omega}{\gamma} \right) \hat{z} + \frac{B_1}{2} \hat{x} \right]}$$

$$B_{eff} = \left(B_0 - \frac{\omega}{\gamma} \right) \hat{z} + \frac{B_1}{2} \hat{x}$$

To QM: irradiating a spin ensemble in lab frame

$$\cancel{i\hbar [\hat{H}, \rho} \quad i\hbar \dot{\rho} = [\hat{H}, \rho]$$

$$\hat{H} = -\vec{M} \cdot \vec{B} = -\gamma \hbar \left[B_0 \hat{S}_z + \cancel{\tfrac{B_1}{2}} B_1 c(\omega t) S_x \right]$$



$B_1 c(\omega t) S_x$ has two components

$$\frac{B_1}{2} \left[S_x c(\omega t) + S_y s(\omega t) \right]$$

$$+ \frac{B_1}{2} \left[S_x c(-\cancel{\omega t}) + S_y \cancel{s(-\omega t)} \right]$$

$$\therefore \quad H = -\gamma \hbar \left[B_0 S_z + \frac{B_1}{2} \left[S_x c(\omega t) + S_y s(\omega t) \right] \right]$$

$$\omega_1 = \frac{\gamma B_1}{2}$$

$$\boxed{H = -\hbar \omega_0 S_z - \hbar \omega_1 \left[S_x c(\omega t) + S_y s(\omega t) \right]}$$

$H = -\hbar\omega_0 S_z - \hbar\omega_1 [\underbrace{S_x c(\omega t) + S_y s(\omega t)}_{e^{-i\omega t S_z} S_x e^{i\omega t S_z}}]$ $\quad \omega t = \phi$

$i\hbar\dot{\rho}$

$\Rightarrow i\hbar\dot{\rho} = [H\rho - \rho H] =$

$$\boxed{\begin{aligned} i\hbar\dot{\rho} &= \underbrace{\boxed{e^{-i\omega t S_z}[-\hbar\omega_0 S_z - \hbar\omega_1 S_x] e^{i\omega t S_z} \rho}}_{H\rho} \\ &- \underbrace{\boxed{\rho\, e^{-i\omega t S_z}[-\hbar\omega_0 S_z - \hbar\omega_1 S_x] e^{i\omega t S_z}}}_{\rho H} \end{aligned}}$$

$e^* S_z e^*$

rotation about z for S_z trivial

Let $\rho_r = e^{i\omega t S_z} \rho e^{-i\omega t S_z}$: rotation about $\hat{z}$ @ rate $(\omega t) = \phi$

& now in rotating frame

$i\hbar\dot{\rho}_r$

$\vdots$

$i\hbar\dot{\rho}_r = [H_r, \rho_r]$ $\quad \boxed{H_r = -\hbar(\omega_0 - \omega)S_z - \hbar\omega_1 S_x}$

& the field b_0 was reduced by $\frac{\omega}{\gamma}$, & b_1 looks static

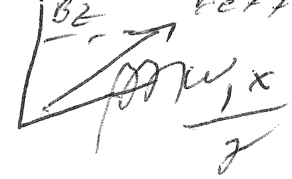
with Pauli matrices behave as:

$$\boxed{\begin{aligned} a_x(t) &= -\Delta\omega a_y \\ a_y(t) &= \Delta\omega a_x + \omega_1 a_z \\ a_z(t) &= -\omega_1 a_y \end{aligned}}$$

$\Delta\omega = \omega_1 - \omega$ = offset

$\omega_1 = \frac{\gamma b_1}{2}$

I-15) $\vec{B}_{eff} = \frac{1}{\gamma}(\Delta\omega_1 \hat{z} + \omega_1 \hat{x})$

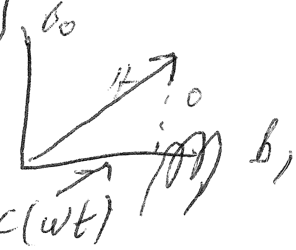


$i\hbar\dot{\rho} = [\hat{H}, \rho] \overset{\text{Soln}}{\Rightarrow} \rho(t) = e^{-iHt/\hbar}\rho(0)e^{iHt/\hbar}$

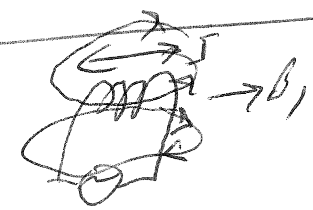
unitary op's again

$\rho(0)$ = time indep Dens Matr
= State at time = 0

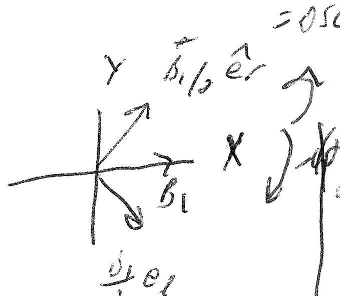
$$\boxed{\hat{H} = -\vec{\mu}\cdot\vec{B} = -\gamma\hbar B_0 S_z - \underbrace{\gamma\hbar B_1 c(\omega t) S_x}_{\text{new time dep term}}}$$



Review I.5 $I = I_0 c(\omega t)$
= oscillating current



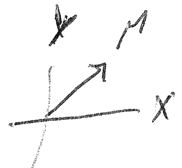
$\Rightarrow B = B_1 c(\omega t)\hat{x}$



2 rotating components opposite direction
~~Drop $+\omega$~~ Drop $-\omega$

The behavior of $\vec{M}$:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{B}$$
$$= \gamma\vec{M} \times (B_0\hat{z} + \frac{B_1}{2}\hat{e}_r)$$



$$= \gamma\vec{M} \times (B_0\hat{z} + \frac{B_1}{2}[c(\omega t)\hat{x} + s(\omega t)\hat{y}])$$

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \left(B_0 \hat{z} + \frac{B_1}{2}\left[C(\omega t)\hat{x} + S(\omega t)\hat{y} \right] \right)$$

Go into rotating frame at rate $B_1(\omega) \Rightarrow \therefore B_1$ looks static

$$\frac{\delta\vec{M}}{\delta t} = \gamma\vec{M}\left[\left(B_0 - \frac{\omega}{\gamma}\right)\hat{z} + \frac{B_1}{2}\hat{x} \right]$$

$B = \frac{\omega}{\gamma}$

Note in rot frame

$$\frac{dM}{dt} = \gamma\vec{M} \times \left[\left(B_0 - \frac{\gamma\omega}{\gamma}\right)\hat{z} + \frac{B_1}{2}\left[C((\omega-\omega)\times t)\hat{x} + S((\omega-\omega)t)\hat{y} \right] \right]$$

$$= \gamma\vec{M} \times \left[B_0 - \frac{\omega}{\gamma} \right]\hat{z} + \frac{B_1}{2}\left[C(0)\hat{x} + S(0)\hat{y} \right]$$

$$= \gamma\hat{M} \times \left[B_0 - \frac{\omega}{\gamma} \right]\hat{z} + \frac{B_1}{2}\left[C(0)\hat{x} + S(0)\hat{y} \right]$$

B_0

$$\star \boxed{\frac{\delta\vec{M}}{\delta t} = \gamma M \times \left[\left(B_0 - \frac{\omega}{\gamma}\right)\hat{z} + \frac{B_1}{2}\hat{x} \right]}$$

$$\frac{\delta\vec{M}}{\delta t} = \vec{M} \times \left[(\gamma B_0 - \omega) + \frac{\gamma B_1}{2}\hat{x} \right]$$

$\omega_1 = \frac{\gamma B_1}{2}$

$$\star \boxed{\frac{\delta\vec{M}}{dt} = \vec{M} \times \left[(\omega_0 - \omega)\hat{z} + \frac{\omega_1}{2}\hat{x} \right]}$$

$\omega_0 - \omega = \text{offset}$

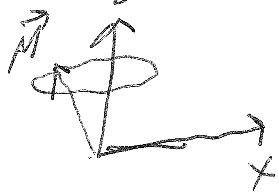
In the rotating frame there is precession of $\vec{M}$ about:

$$\boxed{\vec{B}_{eff} = \frac{B_1}{2}\hat{x} + \left(B_0 - \frac{\omega}{\gamma}\right)\hat{z}}$$

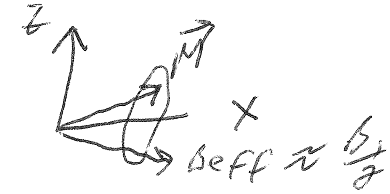
$\omega = \gamma B$

when $B = B_0$

$B_{eff} = B_1/2$



$B_{eff} \approx B_0 - \frac{\omega}{\gamma}$



I-1(b) Calculate the EV's & ES's for:

$$\boxed{H_r = -\Delta w I_z - I_x w_1}$$

$$\boxed{\Delta w = w_0 - w \quad\quad w_1 = \frac{\gamma B_1}{2}}$$

offset in rotating frame

$$I_z = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad I_x = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$\cancel{H_r = -\Delta w} \quad H_r = -\frac{\Delta w}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \frac{1}{2}w_1\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$H_r = -\frac{1}{2}\begin{pmatrix}\Delta w & w_1\\ w_1 & -\Delta w\end{pmatrix}$$

assume $EV = K \quad H\Psi = K\Psi$

Then $|H\Psi - K\Psi| = 0$

$$\det = \begin{vmatrix}-\frac{1}{2}\Delta w - K & \frac{w_1}{2}\\ \frac{w_1}{2} & +\frac{\Delta w}{2} - K\end{vmatrix} = 0$$

$$\left(-\frac{\Delta w}{2} - K\right)\left(\frac{\Delta w}{2} - K\right) - \frac{w_1^2}{4} = 0$$

$$= \left(-\frac{\Delta w}{2}\left(\frac{\Delta w}{2}\right)\right) - \left(\frac{K\Delta w}{2}\right) + \frac{K\Delta w}{2} + K^2 - \frac{w_1^2}{4} = 0$$

$$= K^2 - \frac{\Delta w^2}{4} - \frac{w_1^2}{4} = 0 \Rightarrow K^2 = \frac{\Delta w^2}{4} + \frac{w_1^2}{4}$$

$$K = \pm\frac{1}{2}\sqrt{\Delta w^2 + w_1^2} \quad \Big\} \text{ The EVs}$$

$$\boxed{K = \frac{1}{2}\sqrt{\Delta w^2 + w_1^2}}$$

L

$$\begin{array}{ccc} & |\alpha\rangle & |\beta\rangle \\ \langle\alpha| & \\ \langle\beta| & \end{array} \begin{pmatrix} -\frac{1}{2}\Delta w - (K) & +\frac{w_1}{2} \\ \frac{w_1}{2} & \frac{\Delta w}{2} + K \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0 \quad \left\{ \begin{array}{l} \left(-\frac{\Delta w}{2} - K\right) C_1 + \frac{w_1}{2} C_2 = 0 \\ \frac{w_1}{2} C_1 + \left(\frac{\Delta w}{2} + K\right) C_2 = 0 \end{array} \right.$$

Let $A = -\frac{\Delta w}{2} - K$

$B = \frac{w_1}{2}$

$$C_1 \frac{w_1}{2} = \left(-\frac{\Delta w}{2} - \frac{1}{2}\sqrt{\Delta w^2 + w_1^2}\right) C_2$$

$$C_1 = \left(\frac{-\Delta w}{w_1} - \frac{\sqrt{\Delta w^2 + w_1^2}}{w_1}\right) C_2$$

$$C_1^2 + C_2^2 = 1$$

$$\left(\frac{-\Delta w}{w_1} - \frac{\sqrt{\Delta w^2 + w_1^2}}{w_1}\right)\left(\frac{-\Delta w}{w_1} - \frac{\sqrt{\Delta w^2 + w_1^2}}{w_1}\right) C_2^2 + C_2^2(\quad) = 1$$

$$C_1 \cancel{A} = \begin{pmatrix} A & B \\ B & -A \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0 \quad \left\{ \begin{array}{l} AC_1 + BC_2 = 0 \\ BC_1 - AC_2 = 0 \end{array} \right.$$

$$BC_1 = AC_2 \qquad C_1 = \frac{AC_2}{B}$$

$$C_1^2 + C_2^2 = 1$$

$$\left(\frac{A^2}{B^2}\right) C_2^2 + C^2 = 1$$

$$C_1 = \frac{A}{B}\sqrt{1 + \frac{B^2}{A^2}}$$

$$C_2^2 \left(1 + \frac{A^2}{B^2}\right) = 1$$

$$\therefore C^2 = \frac{1}{1 + \frac{A^2}{B^2}} = 1 + \frac{B^2}{A^2}$$

$$C_2 = \sqrt{1 + \frac{B^2}{A^2}}$$

$$\therefore C_1 =$$

$$\begin{pmatrix} -\frac{1}{2}\Delta w - K & \frac{w_1}{2} \\ \frac{w_1}{2} & \frac{\Delta w}{2} + K \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$$K = \frac{1}{2}\sqrt{\Delta w^2 + w_1^2} = \frac{1}{2} w_{eff}^2 ?$$

$$A = -\frac{1}{2}\Delta w - K$$

$$\begin{pmatrix} A & B \\ B & -A \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{cases} Ac_1 + Bc_2 = 0 \\ Bc_1 - Ac_2 = 0 \end{cases}$$

$$c_1^2 + c_2^2 = 1$$

$$Ac_1 = -Bc_2 \qquad c_1 = \frac{-Bc_2}{A}$$

$$\frac{A^2}{B^2}c_2^2 + c_2^2 = 1$$

$$Bc_1 = Ac_2 \qquad c_1 = \frac{A}{B}c_2$$

$$c_2^2\left(1 + \frac{A^2}{B^2}\right) = 1$$

$$\therefore c_2^2 = \frac{1}{1 + \frac{A^2}{B^2}} = \frac{\frac{B^2}{A^2}}{\frac{B^2}{A^2} + 1}$$

$$\therefore c_1 = \frac{A}{B}\sqrt{\frac{1}{1 + \frac{A^2}{B^2}}} \qquad c_2 = \sqrt{\frac{1}{1 + \frac{A^2}{B^2}}}$$

$$\therefore K = \frac{1}{2}\sqrt{\Delta w^2 + w_1^2}$$

$$\boxed{\begin{gathered} \psi = c_1|\alpha\beta\rangle + c_2|\beta\alpha\rangle \\ \psi = \frac{A}{B}\sqrt{\frac{1}{1 + \frac{A^2}{B^2}}}|\alpha\beta\rangle + \sqrt{\frac{1}{1 + \frac{A^2}{B^2}}}|\beta\alpha\rangle \end{gathered}}$$

$$A = -\frac{1}{2}\Delta w - \frac{1}{2}\sqrt{\Delta w^2 + w_1^2}$$

$$B = \frac{w_1}{2}$$

$$K = -\frac{1}{2}\sqrt{\Delta w_1^2 + w_1^2}$$

$$\begin{pmatrix} \frac{-\Delta w_1}{2} - K & \frac{w_1}{2} \\ \frac{w_1}{2} & \frac{\Delta w_1}{2} + K \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} A & B \\ B & -A \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$A = \frac{-\Delta w_1}{2} - K \qquad Ac_1 + Bc_2 = 0 \qquad Bc_1 = Ac_2$$

$$Bc_1 - Ac_2 = 0 \qquad c_1 = \frac{A}{B} c_2$$

$$B = \frac{w_1}{2}$$

K is diff sign

same as before but

$$\psi = c_1 |\alpha\beta\rangle + c_2 |\beta\alpha\rangle \qquad c_1 = \frac{A}{B}\sqrt{\frac{1}{1 + \frac{A^2}{B^2}}}$$

$$c_2 = \sqrt{\frac{1}{1 + \frac{A^2}{B^2}}}$$

$$A = \frac{-\Delta w_1}{2} + \frac{1}{2}\sqrt{\Delta w_1^2 + w_1^2}$$

$$B = \frac{w_1}{2}$$

K

$$K = +\frac{1}{2}\sqrt{\Delta w^2 + w_1^2} \quad \text{———} \quad c_1 |\alpha\beta\rangle c_2 |\beta\alpha\rangle$$

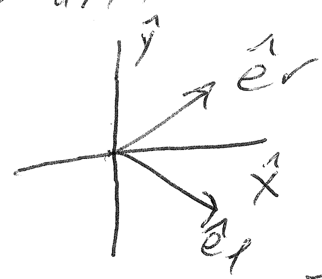
$$K = \frac{-1}{2}\sqrt{\Delta w^2 + w_1^2} \quad \text{———} \quad c_1 |\alpha\beta\rangle + c_2 |\beta\alpha\rangle$$

$$\Delta E = \sqrt{\Delta w^2 + w_1^2} = \text{precession rate about } z$$

from i)

F-17) Show the counter [illegible]

2ω Not sure about this...



$$\frac{d\bar{M}}{dt} = \gamma \vec{M} \times \left[B_0 \hat{z} + \frac{B_1}{2} \hat{e}_r + \frac{B_1}{2} \hat{e}_l \right]$$

Note $B_1 \sim I$

$I = I_0 \, c(\omega t)$

$$= \gamma \hat{M} \times \Big[B_0 \hat{z} + \frac{B_1}{2} \big[c(\omega t) + s(\omega t) \big]$$

$$+ \frac{B_1}{2} \big[c(-\omega t) + s(-\omega t) \big] \Big]$$

$\omega = \gamma B$

~~Going~~ Rotating frame subtract ω

$$\frac{\delta M}{\delta t} = \underbrace{\gamma \hat{M} \times \left[\left(B_0 - \frac{\omega}{\gamma} \right) \hat{z} \right.}_{A} + \frac{B_1}{2} \big[c((\omega-\omega)t)\hat{x} + s((\omega-\omega)t)\hat{y} \big] + \frac{B_1}{2} \big[c((-\omega-\omega)t)\hat{x} - s((-\omega-\omega)t)\hat{y} \big]$$

$$\frac{\delta M}{\delta t} = A + \frac{B_1}{2} \left[c(0)\hat{x} + \cancel{s(0)}\hat{y} \right] + \frac{B_1}{2} \left[c(-2\omega t)\hat{x} - s(-2\omega t)\hat{y} \right]$$

$$\boxed{\frac{\delta M}{\delta t} = A + \frac{B_1}{2}\hat{x} + \frac{B_1}{2} \left[c(-2\omega t)\hat{x} - s(-2\omega t)\hat{y} \right]}$$

$$= c(\theta)\, e^{i\phi I_z} I_x e^{-i\phi I_z} + s\theta\, e \quad I_y\, e$$

$\rho(0) = I_x$

$$= c\theta\left[I_x c\phi - I_y s\phi\right] + s\theta\left[I_y c\phi + I_x s\phi\right]$$

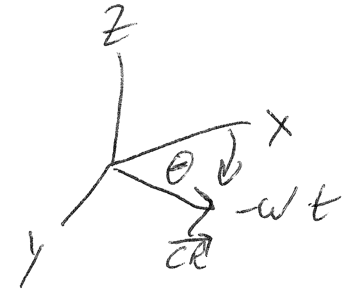
NOTE:

$\vec{CR} = I_x C(\phi) + I_y S(\theta)$

$$= I_x c\theta c\phi + I_x s\theta s\phi$$

$$= I_x c\theta c\phi - I_y c\theta s\phi + I_y s\theta c\phi + I_x s\theta s\phi$$

$$= I_x (c\theta c\phi + s\theta s\phi) + I_y\left[s\theta c\phi - \cancel{c\phi s\theta}\ c\theta s\phi\right]$$



$\theta = -\omega t$

$$= I_x C(\theta - \phi) + I_y\left[s\theta c\phi - c\theta s\phi\right]$$

$$= I_x c(\theta - \phi) + I_y S(\theta - \phi) = \boxed{\text{counter for component}}$$

$$\theta = \phi = -\omega t \Rightarrow I_x C(0) + \cancel{I_y S(0)}^{\,0} = I_x \quad \text{LOOKS STATIONARY}$$

BUT IN OUR ROTATING FRAME w $\phi = +\omega t$, $\theta = -\omega t$

$$= I_x C(\omega t - -\omega t) + I_y S(\omega t - -\omega t)$$

$$\boxed{= I_x \cos(2\omega t) + I_y S(2\omega t)}$$

✓

LOOK UP TRIG IDENTITIES:

$$S(\alpha+\beta) = S\alpha C\beta + C\alpha S\beta$$
$$C(\alpha+\beta) = C\alpha C\beta - S\alpha S\beta$$
$$S(\alpha-\beta) = S\alpha C\beta - C\alpha S\beta$$
$$C(\alpha-\beta) = C\alpha C\beta + S\alpha S\beta$$

$$S(2\theta) = 2 S\theta C\theta$$
$$C(2\theta) = C^2\theta - S^2\theta$$

$$S\alpha C\beta = \frac{S(\alpha+\beta) + S(\alpha-\beta)}{2}$$

$\langle M_y(t)\rangle = \gamma\hbar\langle S_y\rangle$ $\quad$ For $S=\frac{1}{2},\frac{1}{2}$

$S_y = \frac{i}{2}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

$$\boxed{|\psi(t)\rangle = a e^{i\left(\frac{\omega_0 t}{2}+\alpha\right)}|\alpha\rangle + b e^{i\left(-\frac{\omega_0 t}{2}+\beta\right)}|\beta\rangle}$$

Let $A = \frac{\omega_0 t}{2}+\alpha$

$B = -\frac{\omega_0 t}{2}+\beta$

$= a e^{iA}|\alpha\rangle + b e^{iB}|\beta\rangle$

$\langle\psi(t)|\gamma\hbar S_y|\psi(t)\rangle = \langle a e^{-iA} + b e^{-iB}|\gamma\hbar \hat{S}_y|a e^{iA} + b e^{iB}\rangle$

$\frac{\gamma\hbar i}{2}\left(a e^{-iA} + b e^{-iB}\right)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}a e^{iA}\\ b e^{iB}\end{pmatrix} = (\quad)\begin{pmatrix}-b e^{iB}\\ a e^{iA}\end{pmatrix}$

$\phi = \alpha - \beta$

$= \left(-ab e^{-iA}e^{iB} + ab e^{iA-iB}\right)\frac{\gamma\hbar i}{2}$

$A-B = \frac{\omega_0 t}{2}+\alpha-\left(-\frac{\omega_0 t}{2}\right)-\beta$

$= \omega_0 t + \phi$

$= \underbrace{\left(\frac{\gamma\hbar i}{2}\right) ab}_{c}\left[-e^{i(B-A)} + e^{i(A-B)}\right]$

$B-A = -\frac{\omega_0 t}{2}+\beta-\frac{\omega_0 t}{2}-\alpha$

$= -\omega_0 t - \phi$

$= c\left[-e^{i(-\omega_0 t - \phi)} + e^{i(\omega_0 t+\phi)}\right]$

$u = \omega_0 t + \phi$

$= c\left[-e^{-i(u)} + e^{i(u)}\right]$

$= \cancel{c\left[-c(u) + is(u) \pm c(u) \pm is(u)\right]}$

$= c\left[-\left[c(u) - is(u)\right] + c(u) + is(u)\right]$

$= c\left[-c(u) + is(u) + c(u) + is(u)\right]$

$= c\left[2is(u)\right]$

$= c\left[2i^{\circ}S(u)\right]$ $\qquad c = \frac{\gamma\hbar i}{2}ab$

$= \frac{\gamma\hbar i^{\circ}}{2} * 2i^{\circ}\, S(u)\cdot ab$ $\qquad u = \omega_0 t + \phi$

$= -\gamma\hbar\, S(u)\cdot ab$

$= -\gamma\hbar\, S(\omega_0 t + \phi)\cdot ab$

$\boxed{\langle \mu_y \rangle = (-ab\,\gamma\hbar)\sin(\omega_0 t + \phi)}$